Productivity-driven physician scheduling in emergency departments

Fanny Camiat, Maria I. Restrepo, Jean-Marc Chauny, Nadia Lahrichi & Louis-Martin Rousseau

To cite this article: Fanny Camiat, Maria I. Restrepo, Jean-Marc Chauny, Nadia Lahrichi & Louis-Martin Rousseau (2019): Productivity-driven physician scheduling in emergency departments, Health Systems, DOI: 10.1080/20476965.2019.1666036

To link to this article: https://doi.org/10.1080/20476965.2019.1666036

Health Systems
Productivity-driven physician scheduling in emergency departments

Fanny Camiat\textsuperscript{a}, María I. Restrepo\textsuperscript{a}, Jean-Marc Chauny\textsuperscript{b}, Nadia Lahrichi\textsuperscript{a} and Louis-Martin Rousseau\textsuperscript{a}

\textsuperscript{a}Polytechnique Montréal, CIRRELT, Montreal, Quebec, Canada; \textsuperscript{b}Hôpital Sacré-Cœur de Montréal, Université de Montréal, Montreal, Quebec, Canada

**ABSTRACT** The objective of this study is two-fold: to propose an alternative approach for computing the productivity of physicians in emergency departments (EDs); and, to allocate productivity-driven schedules to ED physicians so as to align physician productivity with demand (patient arrivals), without decreasing fairness between physicians, in order to improve patient wait times. Historical data between 2008 and 2017 from the Sacré-Cœur Montreal Hospital ED is analysed and used to predict the demand and to estimate the productivity of each physician. These estimates are incorporated into a mathematical programming model that identifies feasible schedules to physicians that minimise the difference between patients' demand and physicians' productivity, along with the violation of physicians' preferences and fairness in the distribution of shifts. Results on real-world-based data show that when physician productivity is included in the allocation of schedules, demand under-covering is reduced by 10.85\% and the fairness between physicians is maintained. However, physicians' preferences (e.g., sum of the differences between the number of wanted shifts and the number of allocated shifts) deteriorates by 7.61\%. By incorporating the productivity of physicians in the scheduling process, we see a reduction in EDs overcrowding and an improvement in the overall quality of health-care services.

**ARTICLE HISTORY** Received 15 August 2018 Revised 29 June 2019 Accepted 4 September 2019

**KEYWORDS** Physician scheduling; Physicians' productivity; emergency departments

Physician scheduling plays a critical role in Emergency
Departments (ED) planning. The impact of having other specialties simply require physicians “on call” right amount of resources, with the right skills and during their shifts (Carter & Lapierre, 2001). In addition, experience level, can significantly improve the quality of ED physicians must diagnose all types of illnesses while service for patients as well as working conditions for other physicians focus on diseases that are specific to their physicians.

In the province of Quebec, specifically, EDs are known to be the most overcrowded in the world (Cooke et al., 2004). This problem leads to undesirable consequences that degrade Quebec’s health-care system. Some examples of these consequences are: patients leave emergencies without having seen a physician (Derlet & Richards, 2000), physician productivity is decreased, there is extended dissatisfaction and suffering for the patient, and wait times are excessive. The rate of systematic tools to generate schedules, we propose an innovative approach based on physician productivity and visit in 2016 in Quebec was 44%, twice as high than in the estimated hourly demand (patient arrivals) embedded the rest of Canada (Quebec, 2017). Reducing this in a mathematical model. We analyse the ED patient congestion is a high priority to offer better care in EDs arrivals to determine the most accurate demand in Quebec. The government of Quebec highlighted staff forecasting model. Then, we introduce a physicians’ scheduling, among other factors, as a cause this overload productivity index that seeks to reflect the work behaviour (Quebec, 2016). Therefore, to improve this problem, from each physician. This index is based on a heaviness ensuring better planning for ED physicians is one of the classification of each patient (i.e., length of consultation), major challenges to tackle. (Hoot & Aronsky, 2008). This as well as on an estimate of the average number of also true in the United-States (Glass & Anderson, 2018).

Physician planning is a complex task since emergency rooms are open 24 h a day, 7 days a week. ED physicians often work night and weekend shifts, while work regulations. The objective is to minimise the sum of the differences between patient arrivals and physicians’ productivity, along with the violation of physicians’ preferences and fairness in the distribution of shifts. Unfairness between physicians is measured as the sum of differences between the number of day shifts and the number of evening and night shifts allocated to the physicians’ schedules. By minimising this unfairness, the model ensures that the most efficient physicians are not consistently allocated to schedules with high demand. Hence, a rotation of shifts between physicians is guaranteed. This project is motivated by a collaboration with Sacré-Cœur Hospital of Montreal (HSCM). We have received the approval of the ethics department of this Hospital to use anonymous patient data from March 2008 to September 2017, which represent 643,000 records in patient files.

The paper is organised as follows. In Section 2, we review related work on patient arrival prediction. We also present a revision on the computation of physician productivity, as well as physician scheduling. In Section 3, we present the methodology to build physician schedules aimed at reducing overcrowding in EDs. Numerical results are presented and discussed in Section 4. Concluding remarks and future work follow in Section 5.
2. Related work

A general classification of the personnel scheduling process is suggested in Ernst., Jiang., Krishnamoorthy, and Sier (2004). This classification contains several modules starting with the demand modelling to determine staffing requirements and ending with the specification of the work to be performed, over a given planning horizon, by each individual in the workforce. In this section, we present a brief review of recent works addressing the different modules used for the design and allocation of productivity-driven physician schedules in EDs.

2.1. Patient arrivals

The quality of ED services, often measured by waiting time and length of stay, is significantly affected by an accurate prediction of patient arrivals (Xu, Wong, & Chin, 2013), as decisions involving staffing planning and allocation of resources within ED highly depend on these predictions. There has been a significant amount of work done applying different techniques to model and predict the demand in the health-care sector. Time series analysis is among the most popular methods, with applications including the use of exponential smoothing (Boyle et al., 2012; Champion et al., 2007; Jones et al., 2008), autoregressive integrated moving average (ARIMA) models (Boyle et al., 2012; Champion et al., 2007; Sun., Heng, Seow, & Seow, 2009), univariate and multivariate season autoregressive integrated moving average (SARIMA) models (Jones et al., 2008; Kam, Sung, & Park, 2010), multivariate vector autoregressive (VAR) models (Jones et al., 2009), and generalised autoregressive conditional heteroskedasticity (GARCH) models (Jones2002Forecasting, 2002). Linear regression models and nonlinear regression models (Boyle et al., 2012; Xu et al., 2013) have also been proposed as alternatives to predict patient visits to EDs. The previous methods have been successfully used in ED forecasting since they allow seasonal modelling by including variables for the day of week, month of year and holidays. They also allow the identification of repeated patterns in the time-series data. Poisson regression models and artificial neural networks are also among the different alternatives used to predict the demand for ED services. Some applications of these techniques are presented in (Jones et al., 2008; McCarthy et al., 2008; Moineddin, Meaney, Agha, Zagorski, & Glazier, 2011; Xu et al., 2013). The reader is referred to (Wargan, Guidet, Hoang, & Hejblum, 2009) for a review on studies designed to predict patient attendance at EDs or walk-in clinics.

2.2. Productivity

In order to meet patient demand in EDs, it is important to quantify physicians’ productivity so as to know the hospital’s emergency health capacity. Physicians’ productivity is generally defined in the literature by two major indicators: the patients seen per hour (Pt/hr) and the relative value unit (RVU). These indicators study different aspects of physicians’ productivity. The ratio Pt/hr denotes the average number of patients seen by a physician per hour without taking into account patients who were handed over at change of shift (Arya, Salovich, Ohman-Strickland, & Merlin, 2010; Leung et al., 2018). On the other hand, RVU measures estimates of physicians’ effort and practice expense. They reflect the complexity of tasks, thus technical skills, mental effort and psychological stress (Bhargava & Mishra, 2014). Although these indicators were initially proposed as a way of bringing more homogeneity to the health-care reimbursement procedures (Glass & Anderson, 2002), the ratio of Pt/hr and the RVU present some important shortcomings. These are mostly related to changes in medical practice (Storfa & Wilson, 2015). The literature shows various studies to improve physicians’ productivity. The physician payment mechanism, for example, by fee-for-service compensation is evaluated in (Innes et al., 2018). According to (Leung et al., 2018) the use of a physician navigator, a team member that assists a physician in activities to reduce the non-clinical workload during a shift, improves the productivity of ED physicians. In (Arya et al., 2010) it is shown that the utilisation of scribes, a person who assists physicians with the clerical aspects of patient care, can help enhance physician productivity. In this paper, we are
interested in indicators that capture variability in productivity related to the schedules. None of Pt/hr and RVUs take into consideration the ability of physicians to take new patients based on factors related to the type of shift and the type of day. Only (Dula, Dula, Hamrick, & Wood, 2001) have shown that a long sequence of night shifts might decrease the productivity of physicians. We propose to fill this gap and extend the indicator pt/hr to consider alternative factors that might influence productivity (e.g., type of shift, day of the week).

2.3. Physician scheduling

The body of operations research literature directed to ED care services is extensive (Erhard, Schoenfelder, Fügener, productivity (computed as the average ratio of the number & Brunner, 2017). This literature mainly focuses on of patients seen per hour) is proposed to produce an strategic decisions related to regional cov- erage and optimal ED shift schedule. Although these studies show capacity dimensioning for ambulances, and on tactical that aligning physician productivity with patient arrivals decisions associated with physician and nurse scheduling helps to balance staffing costs and to decrease unmet (Hulshof, Kortbeek, Boucherie, Hans, & Bakker, 2012), patient demand in EDs, the differences in performance The importance of physician scheduling lies in the among physicians are not evaluated as the productivity implementation of different staffing levels. These are then ratio is assumed to be the same for each physician. In based on patient arrival rates at different moments within addition, these studies do not incorporate work the day and week to decrease patient waiting times and regulations for the composition of schedules, physicians' reduce the num- ber of patients that leave the ED without availabilities, and fairness in the distribution of shifts, being seen (Green, Soares, Giglio, & Green, 2006). With creating serious limita- tions that degrade the work–life this in mind (El-Rifai, Garaix, Augusto, & Xie, 2015) balance for physi- cians in EDs.

intro- duces a stochastic optimisation model for ED physi- cian scheduling. This model takes into account the stochastic nature of patient arrivals to create physi- cian schedules that respond in a robust way to demand variability. Two studies have already addressed a similar planning horizon including jDj days, where each day d 2 physician scheduling problem for the HSCM ED. In (Gendreau et al., 2006), the problem of constructing physician schedules for emergency rooms demonstrates studies from five dif- ferent hospitals (including HSCM). The authors pro- pose a generic form for the constraints encountered in EDs. In (Beaulieu, Ferland, Gendron, & Michelon, 2000), a mathematical programming model was developed to schedule physicians in ED for a planning horizon of 6 months. Although these studies describe the physician scheduling situation in EDs and show that automated approaches can significantly reduce the time and effort required to construct good-quality divided into two sections: acute care area (A) and physician schedules, the pro- posed approaches do not match physicians' produc- tivity with patient demands, which we believe constitutes a powerful way to reduce patients' waiting time.

Some studies propose incorporating the produc- tivity of physicians in the composition of schedules. An analytic model capable of scheduling providers with different skill profiles and with patients of varying acuity levels is proposed in (Ganguly, Lawrence, & Prather, 2014). In (Savage, Woolford, Weaver, & Wood, 2015), Poisson-based generalised additive models are used to estimate patient arrival rates. A mathematical programming model that incorporates physicians' productivities (e.g., type of shift, day of the week).

3. Problem definition and formulation

The physician scheduling problem in EDs considers a variability. The problem of constructing physician schedules for emergency rooms demonstrates studies from five different hospitals (including HSCM). The authors propose a generic form for the constraints encountered in EDs. In (Beaulieu, Ferland, Gendron, & Michelon, 2000), a mathematical programming model was developed to schedule physicians in ED for a planning horizon of 6 months. Although these studies describe the physician scheduling situation in EDs and show that automated approaches can significantly reduce the time and effort required to construct good-quality divided into two sections: acute care area (A) and physician schedules, the proposed approaches do not match physicians' productivity with patient demands, which we believe constitutes a powerful way to reduce patients' waiting time.

Some studies propose incorporating the productivity of physicians in the composition of schedules. An analytic model capable of scheduling providers with different skill profiles and with patients of varying acuity levels is proposed in (Ganguly, Lawrence, & Prather, 2014). In (Savage, Woolford, Weaver, & Wood, 2015), Poisson-based generalised additive models are used to estimate patient arrival rates. A mathematical programming model that incorporates physicians' productivity (computed as the average ratio of the number & Brunner, 2017). This literature mainly focuses on of patients seen per hour) is proposed to produce an strategic decisions related to regional coverage and optimal ED shift schedule. Although these studies show capacity dimensioning for ambulances, and on tactical that aligning physician productivity with patient arrivals decisions associated with physician and nurse scheduling helps to balance staffing costs and to decrease unmet (Hulshof, Kortbeek, Boucherie, Hans, & Bakker, 2012), patient demand in EDs, the differences in performance The importance of physician scheduling lies in the among physicians are not evaluated as the productivity implementation of different staffing levels. These are then ratio is assumed to be the same for each physician. In based on patient arrival rates at different moments within addition, these studies do not incorporate work the day and week to decrease patient waiting times and regulations for the composition of schedules, physicians' reduce the number of patients that leave the ED without availabilities, and fairness in the distribution of shifts, being seen (Green, Soares, Giglio, & Green, 2006). With creating serious limitations that degrade the work–life this in mind (El-Rifai, Garaix, Augusto, & Xie, 2015) balance for physicians in EDs.

intro- duces a stochastic optimisation model for ED physician scheduling. This model takes into account the stochastic nature of patient arrivals to create physician schedules that respond in a robust way to demand variability. Two studies have already addressed a similar planning horizon including jDj days, where each day d 2 physician scheduling problem for the HSCM ED. In (Gendreau et al., 2006), the problem of constructing physician schedules for emergency rooms demonstrates studies from five different hospitals (including HSCM). The authors propose a generic form for the constraints encountered in EDs. In (Beaulieu, Ferland, Gendron, & Michelon, 2000), a mathematical programming model was developed to schedule physicians in ED for a planning horizon of 6 months. Although these studies describe the physician scheduling situation in EDs and show that automated approaches can significantly reduce the time and effort required to construct good-quality divided into two sections: acute care area (A) and physician schedules, the proposed approaches do not match physicians' productivity with patient demands, which we believe constitutes a powerful way to reduce patients' waiting time.

Some studies propose incorporating the productivity of physicians in the composition of schedules. An analytic model capable of scheduling providers with different skill profiles and with patients of varying acuity levels is proposed in (Ganguly, Lawrence, & Prather, 2014). In (Savage, Woolford, Weaver, & Wood, 2015), Poisson-based generalised additive models are used to estimate patient arrival rates. A mathematical programming model that incorporates physicians' productivity (computed as the average ratio of the number & Brunner, 2017). This literature mainly focuses on of patients seen per hour) is proposed to produce an strategic decisions related to regional coverage and optimal ED shift schedule. Although these studies show capacity dimensioning for ambulances, and on tactical that aligning physician productivity with patient arrivals decisions associated with physician and nurse scheduling helps to balance staffing costs and to decrease unmet (Hulshof, Kortbeek, Boucherie, Hans, & Bakker, 2012), patient demand in EDs, the differences in performance The importance of physician scheduling lies in the among physicians are not evaluated as the productivity implementation of different staffing levels. These are then ratio is assumed to be the same for each physician. In based on patient arrival rates at different moments within addition, these studies do not incorporate work the day and week to decrease patient waiting times and regulations for the composition of schedules, physicians' reduce the number of patients that leave the ED without availabilities, and fairness in the distribution of shifts, being seen (Green, Soares, Giglio, & Green, 2006). With creating serious limitations that degrade the work–life this in mind (El-Rifai, Garaix, Augusto, & Xie, 2015) balance for physicians in EDs.
shifts are used (acute care area shifts $S^A$ and fast-track clinic shifts $S^F$). Each shift $s \in S$ is characterised by a set of attributes, namely: a start time $b_s$, a length $l_s$ and the section of the emergency it covers $e_s$. The demand (given by parameter $d_{sidi}$) denotes the number of patients arriving each day $d \in D$, at each time interval $i \in I_d$, for each emergency section $e \in E$. The objective of the physician scheduling problem in EDs is to allocate feasible schedules to physicians while minimising the under-covering and over-covering of demand (number of new patients arriving at each time period), while also taking into account deviations in physician preferences and the fairness in the distribution of shifts.

In this section, we present the methodology to solve the problem under study. First, we describe the methods adopted to forecast the demand and to compute the physicians’ productivity. Second, we present the notation and formulation of the optimisation model used to generate physician schedules.

3.1. Data description

The HSCM is a university hospital affiliated with the University of Montreal and belonging to one of the five integrated university health and social services centers (CIUSSS) in the city. The hospital can accommodate up to 62,000 patients per year. A total of 35 physicians worked in this hospital in 2017. The demand of the acute care area is covered by six 8-h length shifts starting at 7am, 8am, 3pm, 4pm, 11pm, and 12am. The demand of the fast-track clinic is covered by four 8-h length shifts starting at 7am, 8am, 3pm, and 4pm. Day shifts ($S_D$) start at 7am and 8am, evening shifts ($S_E$) start at 3pm and 4pm, and night shifts ($S_N$) start at 11pm and 12am. This distribution of shifts means that the night acute care physicians assume patients of the fast-track clinic.

Anonymous data were collected from March 2008 to September 2017, including approximately 600,000 entries. Each entry represents a physician consultation.

Variables are divided into two groups. The first group corresponds to the patients’ characteristics and the second group to the consultation characteristics. The variables used in the study are presented in Table 1. The historical information contains 36,021 different worked shifts, representing 288,168 h of work. This information indicates that 45.21% of patients are women and that 54.79% are men. The average age of patients is 51.75 years, 46.66% of patients were treated in the acute care area and 0.27% of patients had a language barrier (i.e., the patient did not speak French or English), and 84% of the records of the historical information correspond to patients who saw only one physician (sometimes a patient is seen by more than one doctor). The proportion of patients in each triage level is presented in Table 2. The lower the level is, the more urgent the patient case is.

3.2. Demand forecasting

The ability to accurately forecast the demand represents an important (and probably one of the first) step for developing robust decision support tools in scheduling and in resource planning for health care in general. In fact, the incorporation of accurate demand forecasts beside other factors including bed availability, laboratory testing, and nursing.

<table>
<thead>
<tr>
<th>Table 1. Types of variables collected.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Patients</td>
</tr>
<tr>
<td>Gender</td>
</tr>
<tr>
<td>Arrival date</td>
</tr>
<tr>
<td>Departure code</td>
</tr>
</tbody>
</table>

$^a$Quebec Nursing Documentation Tool. $^b$Triage level determined by the Canadian Triage and Acuity Scale (CTAS).

F. CAMIAT ET AL.
availability in the different planning levels, i.e., stra-tegic, tactical and operational, could help to improve several operational issues such as physician over-utilisation and long patient waiting times.

To estimate the total number of patients $b_{edi}$ arriving to the emergency room during day $d \in D$ at time interval $i \in I_d$ for emergency type $e \in E$ we used a two-step methodology. First, we forecast the total number of patients $b_{edi}$ arriving at day $d \in D$ for emergency type $e \in E$ (temporal aggregation). This temporal aggregation of the demand was then dis-tributed among the time periods of each day by means of an intra-day distribution model.

The total number of patients $b_{edi}$ was estimated by using a decomposable time series model (Harvey & Peters, 1990) with three main model components: growth, seasonality, and holidays. These components (presented in Equation (1)) represent the growth function ($g_{edi}$) which models non-periodic changes in the value of the time series, the periodic changes function ($s_{edi}$) modelling weekly or yearly seasonality, and the effects of holidays function ($h_{edi}$) including effects from holidays, such as Easter and Christmas day. The error term $\epsilon_{edi}$ represents irregular changes in demand, which are not accommodated by the time series model.

$$b_{edi} = g_{edi} + s_{edi} + h_{edi} + \epsilon_{edi}, \forall e \in E$$ (1)

Equation (1) is estimated with Facebook Prophet (Taylor & Letham, 2018). This tool uses an additive regression model with four components: i) a piecewise linear or logistic growth curve to detect changes in trends by selecting change points from the historical data; ii) a yearly seasonal component modelled using Fourier series; iii) a weekly seasonal component using dummy variables; iv) a user-provided list of relevant holidays. The reader is referred to (Taylor & Letham, 2018) for more information on how Facebook Prophet works.

Let $r_{edi}$ be a parameter denoting the mean of the percentage of the total number of patient arrivals during day $d \in D$ for emergency type $e \in E$ that are allocated to time period $i \in I_d$. This parameter is estimated with nine-year historical data providing information on the number of patients arriving for each emergency type at each time interval of the day. As we assume the month of the year does not have a significant effect in the intra-day distribution model, $r_{edi}$ only varies according to weekdays or weekends. The aggregate estimated demand $\hat{b}_{edi}$ is distributed among the periods of the day by using $r_{edi}$.

Hence, the estimate of the number of patients of type $e \in E$ arriving at time period $i \in I_d$ of day $d \in D$ ($\hat{b}_{edi}$) is given by:

$$\hat{b}_{edi} = \frac{1}{2} r_{edi} \hat{b}_{edi} \forall e \in E, d \in D, i \in I_d$$ (2)

where $\frac{1}{2}$ represents the nearest integer value.

3.3. Physicians’ productivity

The goal of the productivity index is to create a fair indicator that will reflect the capacity of each physician to serve new patients (i.e., patients who were not handed over by another physician). These patients represent 84% of the total time of consults. Hence, treating new patients defines the main task of ED physicians. The HSCM currently allocates patients to physicians based on the acuity level which is determined by the triage level and
which does not reflect the capacity to treat new patients, treated by the physician \( p \in P \) during each shift \( s \in S \) since it is not correlated with consultation length (Yoon, observed in the historical data. This pre- luminary Steiner, & Reinhardt, 2003). That is why we propose a statistical study determined that there is not a significant computing the productivity index by taking into account the effect on the month of the year in this index. However, two components: an estimation of the num- ber of patients this index varies according to the day, the type of shift each physician serves at each time interval of his shift, and the patients’ “heaviness” (i.e., the consultation length).

The patient heaviness is computed for each patient consultation, then a denoted physician later, this latter consult will not be considered. Table 3. Proportion of patients in each category of length Consultations \( \frac{P_{ps}N_{psd}A_{ls}}{\forall p2Pj2J, s2S} \) (4)

\[
\begin{align*}
\sum_{p}^{P} & \frac{P_{ps}^2}{N_{psd}} A_{ls} \cdot \forall p \in P \cdot j \in J \cdot s \in S
\end{align*}
\]

\[P_{ps}^2N_{psd}A_{ls} \cdot \forall p \in P \cdot j \in J \cdot s \in S (4)\]

The most represented category is \( c_3 \) including around 50% of the patients. Categories \( c_1 \) and \( c_2 \) contain 20.87% and 30.01% of the patients, respectively. The consultation length is defined as the duration between the moment the patient enters the physician’s room and the moment he leaves. The consultation time is rounded to the nearest minute. Note that this time is only computed for the first consultation, i.e., if a patient sees another physician later, this latter consult will not be considered.

Each category owns a mean consultation length denoted by \( t_1, t_2 \) and \( t \) as shown in Table 3. Let \( t \) be the mean consultation length for all the consultations (equal to 36 min). With these two values, we construct an intermediate productivity index \( P_{ps}^2 \) based on the number of patients (counted with their weight \( \hat{w} \))

\[
\begin{align*}
\hat{w} & = \frac{1}{4} t \cdot \forall t \in [2, 6] \cdot \forall t_1, t_2 \in [1, 2]
\end{align*}
\]

Table 3. Proportion of patients in each category of consultation with their associated weight.

<table>
<thead>
<tr>
<th>Category</th>
<th>Mean consultation length (t)</th>
<th>Weight (( \hat{w} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>20.87% 10 min</td>
<td>0.3</td>
<td>( c_2 )</td>
</tr>
<tr>
<td>30.01% 22 min</td>
<td>0.6</td>
<td>( c_3 )</td>
</tr>
<tr>
<td>49.12% 55 min</td>
<td>1.5</td>
<td>( c_3 )</td>
</tr>
<tr>
<td>55 min</td>
<td>1.5</td>
<td>( c_3 )</td>
</tr>
<tr>
<td>55 min</td>
<td>1.5</td>
<td>( c_3 )</td>
</tr>
</tbody>
</table>

We develop a mixed-integer programming model that generates a near-optimal schedule responding to different objectives. The objective of the proposed model is to reduce unmet patient demand by matching capacity (physicians’ productivity) with demand (number of patients arriving per hour). Since physician retention is one of the most critical issues facing hospital administrations (Carter & Lapierre, 2001), the model also aims to minimise physicians’ dissatisfaction and unfairness in the distribution of shifts between physicians. Definition of satisfaction may vary from one hospital to the other and from one physician to the other. However, in each hospital, there is a general agreement on what should be considered. As requested by the person in charge of the schedule planning at HSC, physicians’
dissatisfaction is measured as the sum of differences allocated to the schedules. Note that to consider between the number of shifts wanted and the number of individual preferences, we exclude the physicians to shifts allocated. Unfairness between physicians is measured as the sum of differences between the number of day shifts and the number of evening and night shifts apply from the calculation. Physician schedules in EDs are often subject to various constraints. These constraints are divided into four categories (Beaulieu et al., 2000): compulsory constraints, ergonomic constraints, distribution constraints and goal constraints. Compulsory constraints are based on rules that must be absolutely enforced, such as, a rest of 16 h between two consecutive shifts, respecting physicians’ availabilities, and the consideration of physicians’ qualifications to perform certain shifts. For instance, physicians over 50 years-old may not be allocated night shifts. The largest number of constraints are grouped into the category of ergonomic constraints. These constraints aim to improve the quality of the schedules produced by limiting the number of consecutive working days belonging to ergonomic constraints and by enforcing a certain continuity in shifts during the weekend. Distribution constraints limit the number of certain types of shifts allocated to schedules. For instance, each schedule must contain a maximum number of night and weekend shifts allocated within the planning horizon. Finally, goal constraints are based on rules which cannot always be satisfied. For instance, the under-covering and over-covering of patient demand, the under-staffing and over-staffing of the quantity of physicians required to work in each type of shift, and the respect of physicians’ preferences. The reader is referred to 6 for the definition of the decision variables and parameters, as well as the mathematical formulation for the productivity-based physician scheduling problem in EDs.

4. Numerical results
This section presents the computational results obtained after testing the proposed model on real-world-based data. The goal of our study is to improve ED scheduling in order to minimise patient wait times. We want to show that the incorporation of physicians’ productivity is an efficient way to better schedule ED physicians without additional costs. First, we describe the generation of the different instances. Then, we evaluate the differences in schedule quality when using different values for the penalties in the objective function. Finally, we discuss the results.

4.1. Instances generation
To evaluate the ability of our model to cope with demand and productivity variation, we use four scenarios with different weights ($p_1, p_2, p_3$ and $p_4$) associated with each criteria in the objective function. This allows us to find the configuration that better meets the objectives set by HSCM. We remark that the minimisation of under-staffing is the most important objective for HSCM. Hence, this objective is associated with the largest weight in the objective

6 F. CAMIAT ET AL.

function. Scenario 1 reproduces the current situation of the HSCM ED when the coordination between physicians’ productivity and patient demand is not optimised. Scenario 2 characterises the situation when all the goals of the objective function are fairly taken into account. Scenario 3 (resp. Scenario 4) represents the situation when the coordination between physicians’ productivity and patient demand (resp. physicians’ dissatisfactions and unfairness) is prioritised. The different characteristics of the scenarios are summarised in Table 4.

4.2. Accuracy of the demand forecast model and Physicians’ productivity index
In Section 3.2 we introduce a method to forecast the hourly patient demand using Facebook Prophet (Taylor & Letham, 2018). This method takes into account the potential seasonality of the patients’ demand and the effects of holidays. Figure 1 shows there is an upward trend for the patients’ demand in fast-track clinic. It also shows that Christmas and New Year’s day create a punctual impact on the patients’ demand. A weekly trend is also
determined in both fast-track clinic and acute care area.

Figure 2 shows how the forecast demand behaves on its own, compared to the real demand for the first week. This first week represents how a weekly trend is observed in both acute care area and fast-track clinic. We remark that the forecast demand doesn’t exactly predict the sudden variations into 1 day, but only provides the daily trend.

In Section 3.3 we introduced a method to compute the productivity ratio of each physician. These productivity ratios present two particular features which, to the best of our knowledge, have not yet been introduced in the literature. First, we assume that the physicians’ productivity varies from one physician to another. Indeed, as shown in Figure 3, the distribution of the individual mean productivity ratio in both acute care area and fast-track clinic significantly varies among all physicians. Second, we assume that the physicians’ productivity varies within the days of the week.

In Figure 4, we present the weekly variation of the average of the estimate Productivity ratio but also the weekly variation of the average of the physicians’ productivity. The Productivity ratio fits with this productivity in both acute care area and fast-track clinic that proves the relevance of the Productivity ratio. In Table 4. Scenarios.

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Weight for under-staffing ($p_1$)</th>
<th>Weight for over-covering and under-covering ($p_2$)</th>
<th>Weight for unfairness physicians’ dissatisfactions ($p_3$)</th>
<th>Weight for over-covering and under-covering ($p_4$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>100</td>
<td>1</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>S2</td>
<td>100</td>
<td>0</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>S3</td>
<td>100</td>
<td>10</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>S4</td>
<td>100</td>
<td>1</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

We use a planning horizon of 13 weeks as in HSCM.
acute care area, the productivity of physicians during the night shift is, on average, higher than the productivity during the day and the evening shifts. In general, physicians’ productivity is higher during the weekend than during a weekday for both acute care area and fast-track clinic. The productivity in acute care area is not significantly affected by the weekdays. However, we observed a significant difference in the
Figure 1. Seasonality of patients' demand.
productivity of physicians for the fast-track clinic during all days.

4.3. Results

This section presents the results of our study. First, we compare the results of the four scenarios by analysing, from a broad perspective, the values for the different

Figure 2. Comparison of patients’ arrival in the acute care area and in the fast-track clinic per day of the first week.
among physicians and, finally, the seventh category
denotes the performance in respecting physi- cians’
preferences. Results in Table 5 (part 6) does not include
the physicians with individual preferences on night shifts.
Table 5 presents the results of each category for each
scenario.

Scenario 1 reproduces the current schedule of the
HSCM ED. In this case, even though the coordination

We define seven categories of indicators to deter-
mine the performance of each scenario. The first cate-
gory measures the total under-staffling. Categories two and
three measure the under-covering of patients’ demand in
acute care area and fast-track clinic, respectively. Categories four and five measure the over-
covering of patients’ demand in acute care area and fast- track clinic, respectively. The sixth category measures the fairness
between physicians’ productivity and patients’ demand is
not optimised, the fairness in the distribu-
tion of shifts
and physicians’ preferences is optimised. While the values
for fairness and for physicians’ pre-
ferences are the best
among all scenarios, the values related to the
under-covering and over-covering of patients’ demand
are, indeed, the worst among all scenarios. As a result, we
analysed the results of the three last scenarios in order to

The computational experiments were performed on
a Mac OS X operating system, 16 GB of RAM and 1
processor Intel Core i7 running at 2.5 GHz. The algo-

The objectives. Then we analyse, in detail, the distribution of
the differences between patients’ demand and physicians’
productivity throughout the planning horizon.

The first category measures the total under-staffling. Categories two and
three measure the under-covering of patients’ demand in
acute care area and fast-track clinic, respectively. Categories four and five measure the over-
covering of patients’ demand in acute care area and fast- track clinic, respectively. The sixth category measures the fairness

between physicians’ productivity and patients’ demand is
not optimised, the fairness in the distribu-
tion of shifts
and physicians’ preferences is optimised. While the values
for fairness and for physicians’ pre-
ferences are the best
among all scenarios, the values related to the
under-covering and over-covering of patients’ demand
are, indeed, the worst among all scenarios. As a result, we
analysed the results of the three last scenarios in order to

The computational experiments were performed on
a Mac OS X operating system, 16 GB of RAM and 1
processor Intel Core i7 running at 2.5 GHz. The algo-
rithm to solve the problem was implemented in the Julia
programming language (JOC, 2015). The instances were
solved with CPLEX version 12.8.0.0. A relative gap toler-
ance of 5% was set as a stopping criterion for solving the
MILP with CPLEX.

Table 5 presents the results of each category for each
scenario.

Scenario 1 reproduces the current schedule of the
HSCM ED. In this case, even though the coordination
improve under-covering and over-covering of demand without degrading the number of under-staffed shifts, the fairness between physicians, and the respect of physicians’ preferences.

4.4. Total under-staffing

All three scenarios ($S_2$, $S_3$ and $S_4$) present the same values for under-staffing, as the large weight given to $P_1$ ensures minimising under-staffing as much as possible.

4.5. Under-covering and over-covering of patients’ demand

Scenarios 2 and 3 present similar results regarding the under-covering and over-covering of patients’ demand. The percentage of uncovered demand in Scenario 4 is reduced when compared to Scenario 1.
However, this percentage is slightly larger than the one obtained for Scenarios 2 and 3.

4.6. Difference between the number of day shifts and the number of evening and night shifts

The difference between the number of day shifts and the number of evening and night shifts allocated to physicians gives an idea of the fairness in the distribution of shifts between them. Computational results show that schedules are more fair when Scenarios 1 and 4 are used to solve the problem, rather than using Scenarios 2 and 3.

4.7. Difference between the number of wanted shifts and the number of allocated shifts

The differences between the number of wanted shifts and the number of allocated shifts (corresponding to the number of unwanted shifts) denote the respect of physicians’ preferences. Scenario 4 appears to be the best option for the number of unwanted allocated shifts. On the contrary, Scenarios 2 and 3 are simply unacceptable as the number of unwanted shifts.

Table 6. Improvement of the objectives functions. doubles.

<table>
<thead>
<tr>
<th>Objectives Improvement Scenario 4 seems to be the scenario that responds</th>
<th>Number of under-staffed shifts 0% best to the objectives defined. Results related to under-covering and over-covering of patients’</th>
</tr>
</thead>
<tbody>
<tr>
<td>% uncovered demand in acute care area</td>
<td>% uncovered demand in fast-track clinic</td>
</tr>
<tr>
<td>Number of shifts creating an unbalance</td>
<td>10.10%</td>
</tr>
<tr>
<td>11.60%</td>
<td>36.36% demand for Scenario 4 are close to the ones for</td>
</tr>
</tbody>
</table>

Number of allocated shifts not wanted −7.61%

HEALTH SYSTEMS 9

Scenarios 2 and 3, where the coordination between productivity and demand is best. Results concerning fairness between physicians are close to the ones in Scenario 1, where only fairness and the respect of physicians’ preferences are optimised. They are also significantly better than those for Scenarios 3 and 4, which are not acceptable as the degradation of these objectives is simply too large. While the results related to the physicians’ preferences are worse than for those in Scenario 1, they are acceptable nonethe- less. We summarise in Table 6 the improvement in the objectives when Scenario 4 is used instead of Scenario 1 (i.e., the current situation). Negative improvements correspond to a degradation in the corresponding objective.

From Table 6 we observe that the improvement in the alignment between productivity and demand, by using Scenario 4, is significant when compared to using Scenario 1. The percentage of uncovered demand is reduced by 10.10% and by 11.60% in

5. acute care area and fast-track clinic, respectively.

Conclusions While the unfairness in the allocation of shifts decreases by 36.36%, unfortunately, the number of unwanted allocated shifts increases by 7.61%. The graphical results of this comparison are presented in Figure 5.

Figure 5 shows how the physicians’ productivity (corresponding to the number of patients the phy- sician can treat) behaves, on average, in acute care area and fast-track clinic under Scenario 1 (i.e., current schedule) and under Scenario 4 (i.e., improved schedule). The productivity of the

This study shows a substantial gap between the cur- rent schedules used at HSC and the timing of patient arrivals. Incorporating physicians’ productivity in the design and allocation of physician schedules allows to better align the patients demand to the availability of physicians. It also offers a cost-free way (i.e., it is not necessary to hire more physicians) to decrease some of the patients’ unmet demands and waiting times without significantly degrading fairness and physi- cians’ preferences.

improved schedule offers better alignment with

We proposed a forecasting model to predict the patient arrival rates when compared to the pro- arrival of patients and a productivity index including ductivity of the current schedule. This improve-

patients’ heaviness (i.e., consultation time), while also ment is particularly visible on the weekend in the taking into account the productivity variations within fast-track clinic. Indeed, in the current schedule,
a week. These parameters are incorporated into an the physicians’ productivity does not follow the optimisation model that seeks to improve the coordination between physicians’ productivity and patient arrivals while including physicians’ preferences, fairness lower than the afternoon, when the demand is between the allocation of schedules for different physicians, and common work rules in the composition of the ED.

Figure 6 shows the results of the alignment schedules. The proposed model can be generalised to between physicians' productivity and patients' other EDs: the constraints included in the mathematical demands in acute care area and fast-track clinic for formulation are common and relevant to most EDs and the first week. Apart from showing that patients' the objective function can be easily adapted to fit different demands are subject to high variations, this figure also shows that the new schedules improve the gap in vivo and adjust our productivity measure with to serve new patients.

Next step should be to test one of our schedules between the patients’ demand and the ED’s capacity feedback from the field.

Figure 5. Results of the alignment between physicians’ productivity and patients demand in acute care area and fast-track clinic.

- Nature et Technologies.

References


Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This work was supported by the Fonds de Recherche du Québec.

Figure 6. Results of the alignment between physicians' fast-track clinic for the first week.


**Appendix**

**Appendix A. Mathematical model**

**Appendix A1. Variables and parameters description**

Let $p$ be covering $e \in 2 \times P$ be non-negative allocated and a binary under-covering to variable $s \in 2 \times S$ of in takes demand day denoting value $d \times D$. Let $b_{p}^{ed}$ represent positive and negative deviations between the number of day shifts and the number of evening and night shifts allocated to physician $p$ respectively. Let $g_{p}^{+}$ and $g_{p}^{-}$ denote the positive and negative differences between the number of shifts wanted and the number of shifts allocated to physician $p$. Let senting the number of missing physicians required to cover shift $s$ during day $d$ be a slack variable repre-

Table A1. List of parameters.

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_{d}$</td>
<td>$\delta d$</td>
<td>Binary Takes value 1 if shift $s$ covers time period $i$, it assumes value 0 otherwise $w_{d}$</td>
</tr>
<tr>
<td>$w_{d}$</td>
<td>Integer</td>
<td>Indicates the day of the week (e.g. Monday, . . . , Sunday) associated with day</td>
</tr>
<tr>
<td>$p$</td>
<td>Float</td>
<td>Productivity index for physician $p$, during day of the week $j$ and shift $s$</td>
</tr>
<tr>
<td>$b_{p}^{ed}$</td>
<td>Integer</td>
<td>Estimated number of patients arriving at time period $i$ in day $d$ for emergency type $e$</td>
</tr>
<tr>
<td>$g_{p}^{+}$</td>
<td>Integer</td>
<td>Number of shifts wanted by physician $p$</td>
</tr>
<tr>
<td>$g_{p}^{-}$</td>
<td>Integer</td>
<td>Number of budgeted physicians to cover shift $s$ during day $d$</td>
</tr>
<tr>
<td>$d_{p}^{e}$</td>
<td>Binary</td>
<td>Takes value 1 if physician $p$ is qualified to work on shift $s$, it assumes value 0 otherwise $d_{p}^{e}$</td>
</tr>
<tr>
<td>$d_{p}^{e}$</td>
<td>Binary</td>
<td>Takes value 1 if physician $p$ is available to work on shift $s$ of day $d$, it assumes value 0 otherwise $s_{p}^{max}$</td>
</tr>
<tr>
<td>$p$</td>
<td>Integer</td>
<td>Maximal number of shifts preferred by physician $p$</td>
</tr>
<tr>
<td>$n_{p}^{max}$</td>
<td>Integer</td>
<td>Maximal number of night shifts preferred by physician $p$</td>
</tr>
</tbody>
</table>
function

**Appendix A2. Mathematical formulation**

The multi-objective function of the model (5) minimizes under-staffing, under-covering, and over-covering of the patients’ demand. This objective function also ensures a certain fairness, minimizing the differences between the number of day shifts and the number of evening and night shifts allocated to each physician. Objective (5) also ensures the satisfaction of physicians’ preferences, minimizing the difference between the number of shifts wanted and the number of shifts allocated.

\[ \sum_{s \in S} \sum_{d \in D} \sum_{p \in P} \delta_{pds} \cdot b_{pd} \cdot b_{ed} \cdot b_{ad} \geq p_{j} \]

\[ \sum_{s \in S} \sum_{d \in D} \sum_{p \in P} \delta_{pds} \cdot b_{pd} \cdot b_{ed} \cdot b_{ad} \geq p_{j} \]

\[ \sum_{s \in S} \sum_{d \in D} \sum_{p \in P} \delta_{pds} \cdot b_{pd} \cdot b_{ed} \cdot b_{ad} \geq p_{j} \]

\[ \sum_{s \in S} \sum_{d \in D} \sum_{p \in P} \delta_{pds} \cdot b_{pd} \cdot b_{ed} \cdot b_{ad} \geq p_{j} \]

The productivity-driven physician scheduling model is subject to the following constraints:

**Compulsory constraints:** Constraints (A.2) guarantee that each physician is allocated a maximum of one shift per day.

\[ \sum_{s \in S} x_{pds} \leq 1 \forall p \in P, d \in D \] (A.2)

Constraints (A.3) and (A.4) ensure a minimum rest time of 16 h between two consecutive shifts.

\[ \sum_{s \in S} x_{pds} \leq \sum_{s \in S} x_{pds} \]

\[ \sum_{s \in S} x_{pds} \leq \sum_{s \in S} x_{pds} \]

Constraints (A.5) ensure that the availabilities for each physician are respected.

\[ x_{pds} \delta_{pds} \leq 1 \forall p \in P, d \in D, s \in S \] (A.5)

Constraints (A.6) ensure that physician qualifications to perform certain shifts are respected.

\[ x_{pds} \delta_{pds} \leq 1 \forall p \in P, d \in D, s \in S \] (A.6)

**Ergonomic constraints:**

Constraints (A.7) ensure that after a physician ends a night shift at day \( d \) on \( s \), this physician is not allowed to work another night shift the day after.

\[ \sum_{s \in S} \delta_{pds} \cdot x_{pds} \cdot x_{pds} \cdot \delta_{pds} \cdot \delta_{pds} \cdot \delta_{pds} \geq 1 \forall p \in P, d \in D, j \in D \] (A.7)
Constraints (A.7) ensure that after a physician ends a night shift at day $d \geq 1$ (which means the physician starts his shift the day before), the physician is not allowed to work a day or an evening shift the day after.

$$\sum_{s \in S \cap D} x_{pds} \geq \sum_{s \in S \cap D} x_{sds}$$

$$x_{p0d} \geq 1, \forall p \in P, d \in D$$

Notably, constraints (A.7) and (A.8) ensure together that a day-off is allocated after a sequence of night shifts.

Constraints (A.9) prevent isolated nights at each physician schedule.

$$\sum_{s \in S \cap D} x_{pd} \geq 1, \forall p \in P, d \in D$$

HEALTH SYSTEMS 13

Constraints (A.10) set the maximum number of consecutive working days that can be allocated to a physician.

$$\sum_{s \in S \cap D} x_{pd} \leq 3, \forall p \in P, d \in D$$

Constraints (A.11) ensure that each physician is allocated a maximum of five shifts within seven consecutive days.

$$\sum_{s \in S \cap D} x_{pd} \leq 5, \forall p \in P, d \in D$$

Constraints (A.12) set the maximum number of consecutive working days that can be allocated to a physician.

$$\sum_{s \in S \cap D} x_{pd} \leq 2, \forall p \in P, d \in D$$

Constraints (A.13) guarantee that each physician is allocated a maximum of five shifts within seven consecutive days.

$$\sum_{s \in S \cap D} x_{pd} \leq 5, \forall p \in P, d \in D$$

Constraints (A.14) guarantee that each physician is allocated a maximum of 3-night shifts within seven consecutive days.

$$\sum_{s \in S \cap D} x_{pd} \leq 3, \forall p \in P, d \in D$$

Constraints (A.15) ensure that schedules do not contain two consecutive fast-track clinic shifts.

$$x_{pds} \geq x_{0ds} \geq 1, \forall p \in P, d \in D$$

Constraints (A.16), (A.17), and (A.18) ensure a certain continuity within weekend shifts worked by each physician. We denote by $D_F$ and $D_S$ the subsets of days $D$ including Fridays and Sundays, respectively. If a physician works a day shift on Saturday, a day shift is also worked on Sunday by the same physician. If a physician works an evening shift on Friday, evening shifts are also worked on Saturday and Sunday by the same physician. If a physician works a night shift on Friday, night shifts are also worked on Saturday and Sunday by the same physician.

$$\sum_{s \in S \cap D} x_{pds} \geq 1, \forall s \in s \in S$$

$$x_{p0d} \geq 1, \forall p \in P, d \in D$$

(A.15)
\[ x_{pds} \leq \sum_{s \in S^E} \]

\[ x_{pds} \leq \sum_{\forall p \in P, d \in D^F} [D_{jd} \leq d] \]

(A.17)

\[ \sum_{s \in S^E} \]

\[ x_{pds} \leq \sum_{s \in S^E} \]

\[ x_{pds} \leq \sum_{\forall p \in P, d \in D^F} [D_{jd} \leq d] \]

(A.18)

Constraints (A.19) forbid the allocation of two consecutive weekends to each physician schedule. Constraints (A.20) and (A.21) ensure that each physician does not work more than a maximum desired number of shifts and a maximum desired number of night shifts, respectively.

\[ \delta x \sum_{p \in P, d \in D, f \in F} \beta x_{p, d, f} + 2 \delta \]

\[ \sum_{s \in S^E} \]

\[ x_{pds} \leq \]

\[ \forall p \in P, d \in D, j \in J, g \in G \]

(A.19)

Distribution constraints:

Constraints (A.20) and (A.21) guarantee that each physician does not work more than a maximum desired number of shifts and a maximum desired number of night shifts, respectively.

\[ \sum_{s \in S^E} x_{pds} \leq \]

\[ \forall p \in P \] (A.20)

\[ \sum_{s \in S^E} x_{pds} \leq \]

\[ \forall p \in P \] (A.21)

Constraints (A.22) guarantee that a physician does not work more than \( n^w \) weekends during the time horizon.

\[ \sum_{s \in S^E} \]

\[ x_{pds} \leq \]

\[ \forall p \in P \] (A.22)

Goal constraints:

Constraints (A.23) ensure that the number of physicians working during day \( d \) at shift \( s \) is lower than or equal to \( c_{ds} \).

\[ \sum_{p \in P} \]

\[ x_{pds} \leq \]

\[ c_{ds}, \forall d \in D, s \in S \] (A.23)

Constraints (A.24) ensure that the number of patients treated by physicians during day and evening shifts is equal to the patients’ demand subject to some adjustments related to under-covering and over-covering.

\[ \sum_{s \in S^E} \]

\[ x_{pds} \leq \]

\[ \forall p \in P \] (A.24)

Since there are no physicians allocated to fast-track clinic during night shifts, constraints (A.25) ensure that the number of patients treated by physicians of acute care area in the night shifts is equal to the total patients’ demand in both acute care area and fast-track clinic, subject to some adjustments related to under-covering and over-covering.

\[ \sum_{s \in S^E} \]

\[ x_{pds} \leq \]

\[ \forall p \in P \] (A.25)
\[ s \leq S^N \]
\[ \sum_{p \in P} \]
\[ \delta_{si} \Lambda^{p_{d_{i}d_{i}}} \Lambda x_{p_{d_{i}}} b^{A_{d_{i}}} A_{d_{i}} b^{p_{d_{i}} A_{d_{i}}} \]
\[ \frac{1}{4} \sum_{d_{i} \in D} \]
(A.25)
Constraints (A.26) guarantee that each physician works approximately the same number of day shifts as evening and night shifts.
\[ \sum_{d_{i} \in D} \]
\[ \Lambda^{b_{d_{i}}} \forall d \in D, i \in I \]
\[ \sum_{s \in S^{0}} \]
\[ x_{p_{d_{i}}} s^{p_{d_{i}}} A_{p_{d_{i}}} s^{p_{d_{i}}} \frac{1}{4} \sum_{d_{i} \in D} \]
\[ \sum_{s \in S, i \in S_{i}} \]
Constraints (A.26) guarantee that each physician works approximately the same number of day shifts as evening and night shifts.
\[ \sum_{d_{i} \in D} \]
\[ x_{p_{d_{i}}} \forall p \in P \] (A.26)
Constraints (A.27) guarantee that each physician works approximately the number of shifts wanted.
\[ \sum_{d_{i} \in D} \]
\[ \sum_{s \in S} \]
\[ x_{p_{d_{i}}} g_{p_{d_{i}}} \Lambda g_{p_{d_{i}}} b^{p_{d_{i}}} \frac{1}{4} w_{i}, \forall p \in P \] (A.27)