Modeling a Green Inventory Routing Problem with a Heterogeneous Fleet

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Abstract: This paper introduces a green inventory routing problem with a heterogeneous fleet which extends the conventional inventory routing problem by considering environmental impacts and heterogeneous vehicles. A comprehensive objective is proposed, which minimizes the sum of inventory cost and routing cost, where the latter includes driver wage, vehicle fixed cost, fuel and emission costs, in which fuel consumption and emissions are determined by load, distance, speed and vehicle characteristics. We first construct a mixed-integer program, and then conduct numerical tests to quantify the benefits of using a comprehensive objective and heterogeneous vehicles. Managerial insights are also drawn from parameter analyses.

Keywords: Inventory routing, heterogeneous fleet, fuel consumption, CO_2 emissions, valid inequalities, branch-and-cut

1. INTRODUCTION

Global warming is among the greatest challenges of this century, which is mainly caused by carbon dioxide (CO_2) emissions. To respond to this challenge, the United Nations, the European Union, and many countries have enacted legislations to control CO_2 emissions (Hua *et al.*, 2011). Some companies, like IEKA, HP, IBM and GE, also begin to proactively implement ‘green’ initiatives, such as designing greener products or re-optimizing their supply chain networks (Wang *et al.*, 2011).

Supply chain activities, such as production, transportation and inventory, all emit CO_2. However, transportation is the most visible sector of the supply chain which produces most of the CO_2 (Dekker *et al.*, 2012). The Intergovernmental Panel on Climate Change (IPCC) reports that transportation represents 14% of the greenhouse gases (GHG) emissions by economic sectors in 2010 (Pachauri *et al*., 2014). Since CO_2 is the second-greatest contributor to the GHG emissions

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(the first one is water vapor), reducing emissions (for simplicity, in following sections, when we say "emissions", we specifically refer to "CO_2 emissions") by road freight transport will make a good environmental sense. To this end, some companies in Germany and in the United Kingdom have started to adopt more fuel efficient vehicles such as electric and hybrid vehicles (Browne *et al.*, 2011; Taefi *et al.*, 2013). On the other side, companies can also optimize their operational decisions to curb emissions. Benjaafar *et al.* (2013) suggest that sometimes the second method might provide a greater reduction in emissions with less cost than employing low-energy-consumption technologies.

Moreover, Xiao *et al.* (2012) state that it is the cost of fuel consumption not the travel distance which is the greater concern to transportation companies. Therefore, in recent years, many researchers begin to employ fuel consumption cost as variable transportation cost in their models, trying to simultaneously describe the cost configuration correctly and reduce CO_2 emissions. Up to now, there are several papers that tend to minimize fuel consumption (or CO_2 emissions), or use fuel cost to measure variable transportation cost in vehicle routing problem (VRP) (Suzuki, 2011; Xiao *et al.*, 2012; Zhang *et al.*, 2014; Fukasawa *et al.*, 2015; Xiao and Konak, 2016). There are also researchers who combine environmental effects into supply chain network design (Wang *et al.*, 2011; Elhedhli and Merrick, 2012; Martí *et al.*, 2015; Zhalechian *et al.*, 2016). However, studies considering environmental concern in inventory routing problem (IRP) are scare (Treitl *et al*., 2014; Malekly, 2015). Thus, to enrich the research in this direction, a new IRP variant, i.e., the green IRP with a heterogeneous fleet (GIRP-H), is proposed, where both fuel consumption and CO_2 emissions are considered.

Demir *et al.* (2011) have analyzed several models for estimating fuel consumption for road freight transportation. Their study indicates that fuel consumption is determined by a number of factors, such as distance, load, speed and vehicle characteristics. In this study, we compute fuel consumption based on a comprehensive model of Barth *et al.* (2005), and Barth and Boriboonsomsin (2009). This model has been adopted by Bektaş and Laporte (2011), Franceschetti *et al.* (2013) and Koç *et al.* (2014) in pollution-routing problem. Since fuel consumption is the direct cause of CO_2 emissions (Zhang *et al.*, 2014; Cachon, 2014), we can directly transform the amount of fuel consumption, through multiplying by a coefficient, into that of CO_2 emissions.

The rest of this paper is organized as follows. Section 2 introduces related literatures and clarifies the contributions of this study. Section 3 describes our problem and constructs the mathematical model. In Section 4, we add some valid inequalities to strengthen the model and describe solution method. Computational tests and analyses are conducted in Section 5. Section 6 concludes this paper and suggests future research opportunities.

2. LITERATURE REVIEW

The IRP is to determine simultaneously the optimal inventory strategy and vehicle scheduling, thereby minimizing the supply chain system's total (or average) cost. It is first introduced by Bell *et al.* (1983), since then the academic world has conducted extensive research on it. It can be classified according to following criteria: time horizon, demand pattern, supply chain topology, inventory strategy and vehicle fleet (Andersson *et al.*, 2010). Based on different assumptions, some other kinds of IRP can also be defined, such as the IRP with a single product (Zhao *et al.*, 2007) or with multiple products (Coelho and Laporte, 2013a). Interested readers are recommended to review papers by Moin and Salhi (2007), Andersson *et al.* (2010), and Coelho *et al.* (2013). In this section, we only present the studies in terms of time horizon, which is more closely related to our research.

In terms of time horizon, IRP can be divided into two categories: infinite time horizon IRP and finite time horizon IRP (sing period or multiple periods). The objective of infinite time horizon IRP is to minimize the system's average cost by determining optimal replenishment intervals, product quantities delivered and vehicle routes. For finite time horizon IRP, we need to determine the customer sets visited in each period and corresponding product quantities delivered, as well as vehicle routes, in order to minimize the system's total cost.

To solve infinite time horizon IRP efficiently, several policies are introduced, such as fixed-partition policies (FPP) and power-of-two (POT) policy. Under the FPP, retailers are partitioned into different regions. Each time if a retailer in a specific region is served, then the same vehicle must visit all other retailers in the same region (Anily and Federgruen 1990, 1993; Anily and Bramel, 2004). Under the POT policy, retailers' replenishment intervals are limited to power-of-two multiples of a base planning period (Viswanathan and Mathur, 1997; Zhao *et al.*, 2008).

For most companies, logistics planning requires some changes after running for a period of time due to variations in demands and production plans. In this circumstance, infinite time horizon IRP is not applicable. Therefore, recently many researchers begin to study finite time horizon IRP (Archetti *et al.*, 2007; Mion *et al.*, 2011; Coelho and Laporte, 2013; Adulyasak *et al.*, 2013, Desaulniers, *et al.*, 2015). In this study, a multi-period IRP considering environmental implications is investigated.

In traditional supply chain system, companies and researchers only care about profits, costs and service levels. However, with increasing environmental pressures, many begin to consider fuel consumption or CO_2 emissions in their problems. Although in recent years, a few scholars start to consider environmental issues in IRP, they usually simplify the calculation of fuel consumption or emissions, which is not accurate and realistic. For example, Alkawaleet *et al.* (2014), and Mirzapour and Rekik (2014) only consider travel distance in computing emissions. In our study, a comprehensive model is constructed, where fuel consumption and CO_2 emissions are influenced not only by distance, but also by load, speed and vehicle characteristics.

Another key aspect of our work is that a heterogeneous fleet is used, because in real-world distribution problems, deliveries are usually implemented by several types of vehicles (Hoff *et al.*, 2010; Koç *et al.*, 2014). There are two types of problems belonging to this category: the fleet size and mix VRP and the heterogeneous VRP. The distinction is that the former one usually assumes the number of vehicles to be unlimited; however, the latter often works with a limited fleet. Although a few papers that study green IRP calculate fuel consumption as we will do (Al Shamsi *et al.*, 2014; Treitl *et al.*, 2014; Malekly, 2015; Soysal *et al.*, 2015, 2016), within our knowledge, the green IRP with a heterogeneous fleet has not yet been studied. Al Shamsi *et al.* (2014) and Malekly (2015) work with 1 vehicle, and the other three papers use a homogeneous fleet. We believe there is merit in investigating heterogeneous green IRP, because it is often difficult for a homogeneous fleet to simultaneously control costs and CO_2 emissions. For instance, as to light duty vehicles, although their fixed transportation cost is low, their limited capacity will lead to more trips and travel distance, which might result in more fuel cost and CO_2 emissions. For medium and heavy duty vehicles, although their travel distance can be lowered, their curb weight and fixed transportation cost are much higher, which may still influence cost and CO_2 emissions. Actually, our numerical tests in later section do demonstrate that a heterogeneous fleet can better balance costs and environmental impacts.

The third aspect of our work is that we develop a comprehensive objective function, to make the problem as realistic as possible. The objective includes the inventory cost, driver wage, vehicle fixed cost, fuel cost and emission cost. In traditional IRP models, the objective is simple, only inventory cost and variable transportation cost (distance-oriented) are included (Archetti *et al.*, 2007; Coelho and Laporte, 2013a, 2013b, 2014).

To conclude, our research contributes to the literature on IRP by (1) introducing a comprehensive IRP objective that accounts for cost, environmental implications and different vehicle types, which is a new IRP variant; (2) analyzing the benefits of using a comprehensive objective function and a heterogeneous fleet, respectively; and (3) performing numerical tests to provide managerial insights.

3. PROBLEM DEFINITION AND MATHEMATICAL FORMULATION

3.1 Problem Definition

We consider an outbound distribution network in which a supplier, denoted by 0, distributes a single product to N geographically dispersed retailers over a finite time horizon $T = \{1,2,\ldots p\}$. In each period, the quantity of products made available at the supplier is r^t. Retailer $i \in V' = \{1,2, \dots N\}$ faces an external demand of d_i^t in period $t \in T$ and keeps the inventory level I_i^t at the end of period t. We assume that in the beginning of the time horizon the initial inventory level at vertex $i \in V = V' \cup \{0\}$ is I_i^0. Both the supplier and retailers incur a unit inventory holding cost h_i per period. Meanwhile, inventories are not permitted to be negative at any vertex. A maximum-level inventory replenishment policy is applied at retailers, which indicates that any quantity of products can be delivered to a retailer as long as the retailer's maximal inventory capacity C_i is not exceeded. There are $M = \{1,2, \dots, |M|\}$ types of vehicles housed at the supplier, and the number of vehicles available for type $m \in M$ is n^m. Q^m is the weight capacity of vehicle type m. In each period, vehicles start their trips from the supplier and return to it after finishing deliveries, and all related trips can be finished in one day. We limit a retailer to be visited by at most one vehicle in each period. The travel distance from vertex i to vertex j is c_{ij}.

The binary variable x_{ij}^{mt} is equal to 1 if and only if a vehicle of type m travels from vertex i to vertex j in period t. Let a_{ij}^{mt} be the product weight carried by vehicle type m between arc (i,j) in period t. v_{ij}^{mt} represents the speed at which vehicle type m travels from vertex i to vertex j in period t. q_i^{mt} is the product weight delivered to retailer i by vehicle type m in period t. y_i^{mt} takes value 1 if and only if vertex i is visited by vehicle type m in period t, and 0 otherwise. The order of events at the supplier and retailers are shown in Figure 1, in which the supplier first receives (or manufactures) r^t and then replenishes retailers, and retailer i first receives $\sum_{m \in M} q_i^{mt}$ and then consumes d_i^t.

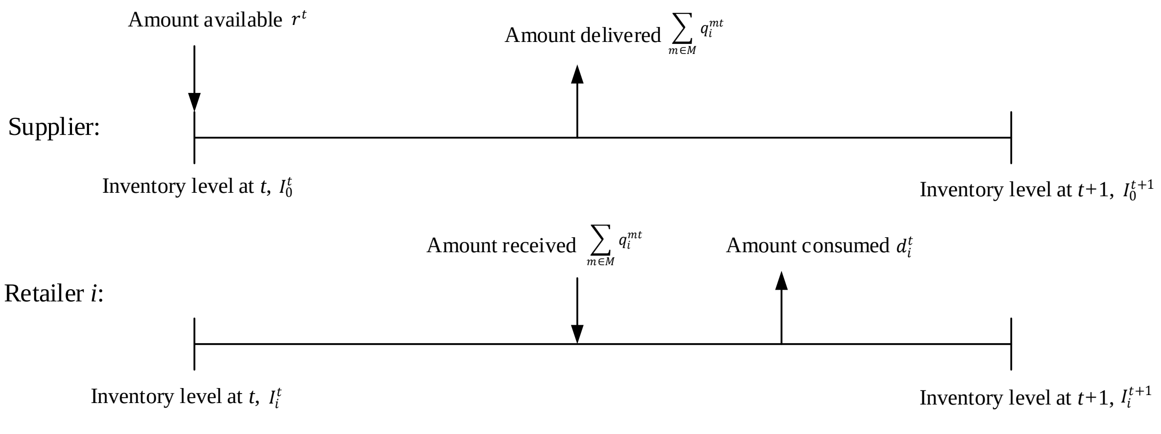

Figure 1 The order of events at the supplier and retailers

3.2 Fuel Consumption Calculation

Based on a comprehensive model of Barth *et al.* (2005), and Barth and Boriboonsomsin

(2009), the fuel consumption F^m (*liters*) of a vehicle of type m over a distance d at a speed of v can be calculated as

$$F^m = \lambda\left(\frac{k^m N^m V^m d}{v} + M^m \gamma^m \alpha d + \beta^m \gamma^m d v^2\right) \tag{1}$$

in which $\lambda = \varepsilon/(\kappa\psi)$, $\gamma^m = 1/(1000 n_{tf}^m \eta)$, $\alpha = \tau + g\sin\theta + gC_r\cos\theta$, $\beta^m = 0.5C_d^m \rho A^m$. M^m represents the total vehicle weight (*kg*), which is the sum of curb weight and payload. Other parameters' definition and typical values can refer to Table 1 and Table 2.

Table 1 Vehicle common parameter definition

Notation	Description	Typical values
ε	Fuel-to-air mass ratio	1
g	Gravitational constant (*m/s*^{2})	9.81
ρ	Air density (*kg/m*^{3})	1.2041
C_r	Coefficient of rolling resistance	0.01
η	Efficiency parameter for diesel engines	0.45
f_c	Unit fuel cost (*£/L*)	0.7382*
f_e	Unit CO_2 emission cost (*£/kg*)	0.248*
f_d	Driver wage (*£/s*)	0.0022
σ	CO_2 Emitted by unit fuel consumption (*kg/L*)	2.669*
κ	Heating value of a typical diesel fuel (*kj/g*)	44
v	Speed (*m/s*)	--
ψ	Conversion factor (*g/s* to *L/s*)	737
θ	Road angle	0
τ	Acceleration (*m/s*^{2})	0
d	Travel distance (*m*)	--

*Source: Cachon (2014), Demir *et al.* (2012), Koç *et al.* (2014) and Soysal *et al.* (2015, 2016).
Other parameters' values can refer to Koç *et al*. (2014).

Table 2 Vehicle specific parameter definition

Notation	Description	Light duty	Medium duty	Heavy duty
w^m	Curb weight (*kg*)	4672	6328	13154
Q^m	Maximum payload (*kg*)	2585	5080	17236
f^m	Vehicle fixed cost (*£/day*)	41.68	59.90	93.92
k^m	Engine friction factor (*kj/rev/L*)	0.25	0.20	0.15
N^m	Engine speed (*rev/s*)	39	33	30.2
V^m	Engine displacement (*L*)	2.77	5.00	6.66
C_d^m	Coefficient of aerodynamics drag	0.6	0.6	0.7
A^m	Frontal surface area (*m*^{2})	9.0	9.0	9.8
n_{tf}^m	Vehicle drive train efficiency	0.40	0.45	0.50

Parameters' values can refer to Koç *et al*. (2014).

We use Equation (1) to plot the fuel consumption of different vehicles with respect to speed, shown in Figure 2, in which vehicles' payloads are set to 75% of their maximum payloads. We find that for each type of vehicles the fuel consumption at a speed of 30 or 40 *km/h* is the least, and that under the same speed different vehicle type has a big difference in fuel consumption. Therefore, it makes sense to consider the influence of speed and vehicle characteristics in GIRP-H.

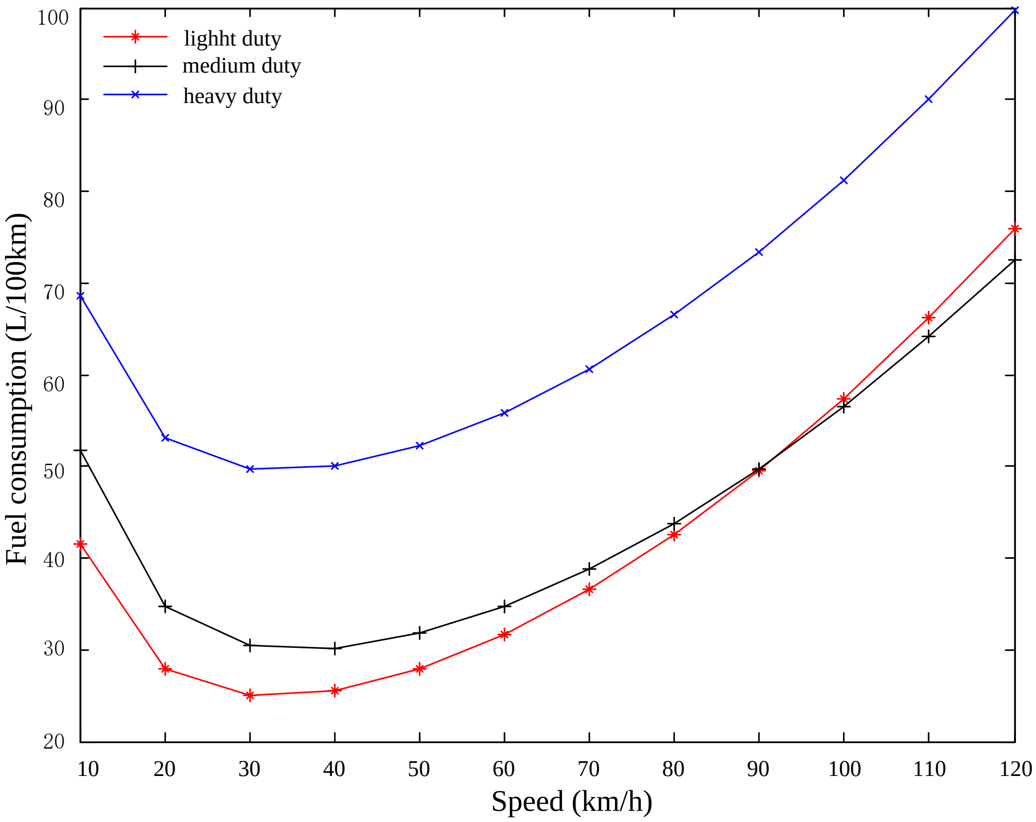

Figure 2 The fuel consumption of each type of vehicles under different speeds

3.3 Mathematical Formulation

The completed formulation for the proposed problem can be represented as follows:

$$\text{Minimize} \quad \sum_{i\in V}\sum_{t\in T} h_i\, I_i^t \tag{2.a}$$

$$+\sum_{i\in V}\sum_{j\in V}\sum_{m\in M}\sum_{t\in T}\left(c_{ij}/v_{ij}^{mt}\right) f_d x_{ij}^{mt} \tag{2.b}$$

$$+\sum_{t\in T}\sum_{m\in M}\sum_{j\in V'} f^m\, x_{0j}^{mt} \tag{2.c}$$

$$+\sum_{i\in V}\sum_{j\in V}\sum_{m\in M}\sum_{t\in T}\lambda\left(\frac{x_{ij}^{mt}k^m N^m V^m c_{ij}}{v_{ij}^{mt}} + \left(w^m x_{ij}^{mt} + a_{ij}^{mt}\right)\gamma^m \alpha c_{ij} + x_{ij}^{mt}\beta^m \gamma^m c_{ij}(v_{ij}^{mt})^2\right)(f_c + f_e\sigma) \tag{2.d}$$

Subject to

$$I_0^t = I_0^{t-1} + r^t - \sum_{i\in V'}\sum_{m\in M} q_i^{mt} \qquad t\in T \tag{3}$$

$$I_i^t = I_i^{t-1} + \sum_{m\in M} q_i^{mt} - d_i^t \qquad i\in V', t\in T \tag{4}$$

$$I_i^t \le C_i \qquad i\in V', t\in T \tag{5}$$

$$\sum_{m\in M} q_i^{mt} \le C_i - I_i^{t-1} \qquad i \in V', t \in T \tag{6}$$

$$q_i^{mt} \le C_i y_i^{mt} \qquad i \in V', m \in M, t \in T \tag{7}$$

$$a_{ij}^{mt} \le Q^m x_{ij}^{mt} \qquad i \in V, j \in V, m \in M, t \in T \tag{8}$$

$$\sum_{i\in V'} x_{0i}^{mt} \le n^m \qquad m \in M, t \in T \tag{9}$$

$$\sum_{m\in M} \sum_{i\in V} x_{ij}^{mt} \le 1 \qquad j \in V', t \in T \tag{10}$$

$$\sum_{j\in V} x_{ij}^{mt} = \sum_{j\in V} x_{ji}^{mt} \qquad i \in V, m \in M, t \in T \tag{11}$$

$$\sum_{j\in V} x_{ij}^{mt} + \sum_{j\in V} x_{ji}^{mt} = 2y_i^{mt} \qquad i \in V, m \in M, t \in T \tag{12}$$

$$\sum_{i\in V} a_{ij}^{mt} - \sum_{i\in V} a_{ji}^{mt} = q_j^{mt} \qquad j \in V', i \ne j, t \in T, m \in M \tag{13}$$

$$x_{ii}^{mt} = 0 \qquad i \in V, t \in T, m \in M \tag{14}$$

$$a_{ij}^{mt} \ge 0 \qquad i \in V, j \in V, m \in M, t \in T \tag{15}$$

$$q_i^{mt} \ge 0 \qquad i \in V', m \in M, t \in T \tag{16}$$

$$I_i^t \ge 0 \qquad i \in V, t \in T \tag{17}$$

$$x_{ij}^{mt} \in \{0,1\} \qquad i \in V, j \in V, m \in M, t \in T \tag{18}$$

$$y_i^{mt} \in \{0,1\} \qquad i \in V, m \in M, t \in T \tag{19}$$

The objective comprises four parts: (2.a) inventory holding cost, (2.b) driver cost, (2.c) vehicles' fixed cost, and (2.d) fuel and CO_2 emission costs. Constraints (3) and (4) are the inventory balance equations at the supplier and retailers respectively. Constraints (5) limit maximal inventory level at retailers. Constraints (6) impose maximum-level inventory replenishment policy. Constraints (7) represent that if no vehicles of type m visit retailer i, then the product quantity delivered to retailer i by vehicle type m is 0. Constraints (8) mean that vehicles' capacities must be respected. Constraints (9) limit the number of vehicles that can be used. Constraints (10) indicate that each retailer can be visited at most once in each period. Constraints (11) are the vehicle balance equations. Constraints (12) are degree constraints. Constraints (13) are the product flow balance equations at retailers and eliminate all subtours. Constraints (14) define impossible arcs. Constraints (15)-(19) enforce non-negativity and integrality conditions on variables.

It is noticed that the objective function involves two nonlinear terms, i.e., equations (2.b) and (2.d). We use the procedure proposed by Bektaş and Laporte (2011) to linearize them, through a discretization of the speed variable v_{ij}^{mt}. We assume that the lower and upper bounds of speed on each arc are l and u respectively, and define a set of speed levels $\mathcal{R} = \{1, 2, \dots, r\}$. Then each $r \in \mathcal{R}$ for a given arc (i, j) corresponds to a speed interval $[l^r,\ u^r]$ with $l^1 = l$ and $u^{|\mathcal{R}|} = u$. Next we compute the average speed as $\bar{v}^r = (l^r + u^r)/2$ for each speed level $r \in \mathcal{R}$. We introduce a binary variable $z_{ij}^{mtr} = 1$ if vehicle type m travels at speed level r on arc (i, j) in period t, and 0 otherwise. Therefore, z_{ij}^{mtr} and x_{ij}^{mt} are linked by the following equation

$$\sum_{r\in\mathcal{R}} z_{ij}^{mtr} = x_{ij}^{mt} \quad i, j \in V, m \in M, t \in T \tag{20}$$

And the linearized model is as follows:

$$\text{Minimize } \sum_{i\in V} \sum_{t\in T} h_i I_i^t \tag{2.a'}$$

$$+ \sum_{i\in V} \sum_{j\in V} \sum_{m\in M} \sum_{t\in T} \sum_{r\in\mathcal{R}} \frac{c_{ij}}{\bar{v}^r} z_{ij}^{mtr} f_d \tag{2.b'}$$

$$+ \sum_{t\in T} \sum_{m\in M} \sum_{j\in V'} f^m x_{0j}^{mt} \tag{2.c'}$$

$$+ \sum_{i\in V} \sum_{j\in V} \sum_{m\in M} \sum_{t\in T} \lambda \left(\sum_{r\in\mathcal{R}} \frac{k^m N^m V^m c_{ij}}{\bar{v}^r} z_{ij}^{mtr} + \left(w^m x_{ij}^{mt} + a_{ij}^{mt}\right) \gamma^m \alpha c_{ij} + \beta^m \gamma^m c_{ij} \left(\sum_{r\in\mathcal{R}} \bar{v}^r\right)^2 z_{ij}^{mtr} \right) (f_c + f_e \sigma) \tag{2.d'}$$

Subject to constraints (3)-(20).

4. VALID INEQUALITIES and SOLUTION METHOD

4.1 Valid Inequalities

Archetti *et al.* (2007) and Coelho and Laporte (2014) propose several classes of valid inequalities for the IRPs. We extend some of them to strengthen our model.

$$y_i^{mt} \le y_0^{mt} \qquad i \in V', m \in M, t \in T \tag{21}$$

Inequalities (21) mean that if no vehicle of type m leaves the supplier, then no vehicle of type m will visit retailers.

Constraints (22) imply that during the time interval $[t_1, t_2]$ retailer i is visited at least the number of times corresponding to the right side of the inequality, in which Q is the maximal value among $Q^1, Q^2, \dots Q^m$, that is, Q= max$\{Q^1, Q^2, \dots Q^m\}$.

$$\sum_{m\in M} \sum_{t'=t_1}^{t_2} y_i^{mt'} \ge \left\lceil \frac{\sum_{t'=t_1}^{t_2} d_i^{t'} - C_i}{\min\{Q, C_i\}} \right\rceil \qquad i \in V', t_1, t_2 \in T, t_2 \ge t_1 \tag{22}$$

We extend inequalities (22) by considering retailer's actual inventory level in the numerator rather than its inventory capacity, which yields inequalities (23):

$$\sum_{m\in M} \sum_{t'=t_1}^{t_2} y_i^{mt'} \ge \frac{\sum_{t'=t_1}^{t_2} d_i^{t'} - I_i^{t_1-1}}{\min\{Q, C_i\}} \qquad i \in V', t_1, t_2 \in T, t_2 \ge t_1 \tag{23}$$

For retailer i, if its inventory at the end of period t_1-1 is enough to satisfy its demands over $[t_1, t_2]$, then it is not mandatory for vehicles to visit the retailer. Otherwise, at least one visit must take place. Consequently, we propose inequalities (24):

$$\sum_{m\in M}\sum_{t'=t_1}^{t_2} y_i^{mt'} \geq \frac{\sum_{t'=t_1}^{t_2} d_i^{t'} - I_i^{t_1-1}}{\sum_{t'=t_1}^{t_2} d_i^{t'}} \qquad i \in V', t_1, t_2 \in T, t_2 \geq t_1 \tag{24}$$

Note that the right side of equation (22) has a difference from those of (23) and (24). In equation (22), we round up the right side, because they are constants. However, this is not permitted in the latter two equations, as their numerators include decision variables and they will become non-linear otherwise. The impact of these inequalities on computational efficiency will be evaluated in later section.

4.2 Solution Method

In order to exactly display the difference between objective functions and the benefits of using a heterogeneous fleet, exact solutions are preferred in this study. Furthermore, as multi-period IRP belongs to a medium-term planning for companies, which will affect their operations for several weeks or months, the quality of solutions is more important than the computation time. Thus, the branch-and-cut algorithm is employed.

In our branch-and-cut algorithm, we first add user cuts (i.e., equations (21)-(24)) to the model at the search tree's root node. Then we use a commercial solver to solve linear programming relaxation problems at each node. If no feasible solution is found for a node's relaxation problem, then this node is pruned and the algorithm starts to check other active nodes based on a best bound strategy; however, if a feasible one is found, the algorithm either updates the incumbent optimal solution (if the feasible solution has no fractional variables, and it is better than the current solution) or adds cuts (if the feasible solution has fractional variables). When new cuts are added to the relaxation model, the model is re-optimized. If all cuts are respected and the current solution of a relaxation problem still includes fractional variables, then the algorithm begins to branch on a fractional variable to produce two new subproblems. This process is reiterated until no active nodes exist.

5. COMPUTATIONAL ANALYSES

The aim of computational tests is fourfold: (1) to test the efficiency of valid inequalities (Section 5.2), (2) to compute the savings that could be achieved by considering a comprehensive objective function over the traditional objective (Section 5.3), (3) to quantify the benefits of applying a heterogeneous fleet (Section 5.4), and (4) to analyze the influence of parameters on key performance indicators (Section 5.5).

The branch-and-cut algorithm is implemented in C++ language using the IBM Concert Technology and CPLEX 12.6 as the solver. 4 threads are used. All computations are executed on a PC with Intel Core i5 Processor (2.3 GHz) and 4 GB memory.

5.1 Instance Data

We conduct our numerical analyses based on the data generated by Coelho and Laporte (2013a) for a single product. We use their data of vertices' coordinates and compute distance as $c_{ij} = 100 \times \left\lfloor\sqrt{(x_{i-}x_j)^2 + (y_{i-}y_j)^2 + 0.5}\right\rfloor$, where (x_i, y_i) are the coordinates of vertex i. We enlarge their product-related data by a factor of 10 for 10 retailers' instances, a factor of 6 for 20 retailers' instances, including r^t, d_i^t, I_i^0 and C_i. Besides, all vertices' inventory holding cost coefficients are reduced by a factor of 10. We also employ their labels for each instance, for example, label "10-1-3-5-2" means that it is the second instance with 10 customers, 1 kind of products, 3 homogeneous vehicles and 5 periods in their study. Since there are 3 types of vehicles in our paper, the label indicates that it is the second instance with 10 customers, 1 kind of products, 3 types of vehicles and 5 periods in our work.

We assume the number of vehicles for each type is unlimited (i.e., $n^m = N, \forall m \in M$), which provides flexibility since it allows the actual number to be determined later. The lower and upper bounds of speed for each type of vehicles are 20 *km/h* and 70 *km/h*, respectively. 5 speed levels are defined, that is, the speed is divided into 5 intervals (i.e., [20, 30], [30, 40], [40, 50], [50, 60], [60, 70]).

5.2 Computational Complexity

To evaluate the influence of valid inequalities on computational efficiency, we perform an analysis based on randomly selected 10 instances.

Table 3 The influence of valid inequalities on the computational time

Case	Instance	Model		Model +(21)		Model +(22)		Model +(23)		Model +(24)		Model +(21)-(24)	
		Time(s)	Gap[a]	Time(s)	Gap	Time(s)	Gap	Time(s)	Gap	Time(s)	Gap	Time(s)	Gap
1	20-1-1-3-1	2558.74	0.00	**529.15**	0.00	**1472.23**	0.00	3600.00	1.93	**544.10**	0.00	**324.41**	0.00
2	20-1-1-3-2	735.48	0.00	**21.84**	0.00	**901.53**	0.00	**943.52**	0.00	**334.21**	0.00	**44.69**	0.00
3	20-1-1-3-3	3600.00[b]	8.69	**3038.80**	**0.00**	**673.94**	**0.00**	**323.27**	**0.00**	**1170.67**	**0.00**	**537.01**	**0.00**
4	20-1-1-3-4	923.49	0.00	**420.39**	0.00	1501.4	0.00	2289.79	0.00	1554.08	0.00	1292.58	0.00
5	20-1-1-3-5	3600.00	5.47	**3353.50**	**0.00**	3600.00	8.86	3600.00	**4.94**	3600.00	**4.69**	3600.00	**1.25**
6	20-1-3-3-1	3600.00	7.76	3600.00	**3.33**	3600.00	**3.37**	3600.00	**1.29**	**3159.38**	**0.00**	**960.55**	**0.00**
7	20-1-3-3-2	3600.00	7.14	3600.00	**0.80**	3600.00	**2.26**	3600.00	**1.43**	3600.00	**2.05**	3600.00	**3.24**
8	20-1-3-3-3	1212.16	0.00	**196.91**	0.00	**315.29**	0.00	**324.00**	0.00	**1027.46**	0.00	**103.40**	0.00
9	20-1-3-3-4	103.32	0.00	**19.68**	0.00	**56.45**	0.00	176.05	0.00	106.20	0.00	**20.94**	0.00
10	20-1-3-3-5	98.36	0.00	**32.25**	0.00	270.52	0.00	148.38	0.00	117.46	0.00	**32.94**	0.00
	Average	2180.72	2.91	**1481.25**	**0.41**	**1599.14**	**1.45**	**1860.50**	**0.96**	**1521.36**	**0.67**	**1051.65**	**0.45**

[a] The percentage gap between the best integer and the lower bound

[b] The algorithm ends after 3600 second time limit

Bold numbers indicate that the computation time or gap is smaller than that without any valid inequalities.

Table 3 demonstrates that all the valid inequalities can improve computational efficiency, equations (21) in particular. The model with all valid inequalities can reduce computation time by 47.50% on average with a smaller gap, compared to the scenario without any valid inequalities. More specifically, for cases 1, 2, 3 and 8, computational time can be reduced by 5 to 15 times. Besides, through observing the computation process, we find that for most instances the branch-and-cut algorithm is able to find satisfying solutions in 5 minutes (the percentage gap between the best integer and the lower bound is less than 2%). Therefore, in realistic applications if decision-makers do not want to spend too much time on computation, they can require the algorithm to end when a preset gap is met. In following analyses, we solve instances with 10 and 20 retailers to optimality.

Moreover, since there are three types of binary variables in our model, i.e., x_{ij}^{mt}, y_i^{mt} and z_{ij}^{mtr}, in the branching phase we consider four methods: branching is conducted in priority on x_{ij}^{mt}, y_i^{mt} and z_{ij}^{mtr} respectively; and let CPLEX automatically decide which variable to branch on. It is observed that the last method works best and the first one (x_{ij}^{mt} in priority) is the worst case.

We also add lazy constraints $\sum_{i\in S}\sum_{j\in S} x_{ij}^{mt} \leq |S|-1,\ m \in M, t \in T, S \in V'$ dynamically during executing the algorithm. However, numerical tests show that they do not help improve computational efficiency. Thus, when executing the branch-and-cut algorithm, we employ user cuts (21)-(24) and cuts generated by CPLEX automatically.

5.3 Impact of Cost Components

We now calculate the cost saving companies can achieve by considering a comprehensive objective. To this end, we compare the proposed objective with the traditional one (Archetti *et al.*, 2007; Coelho and Laporte, 2013a, 2013b, 2014), where the latter is to minimize the sum of inventory holding cost and variable transportation cost, that is,

$$\text{Minimize } \sum_{i\in V}\sum_{t\in T} h_i I_i^t + \sum_{i\in V}\sum_{j\in V}\sum_{m\in M}\sum_{t\in T} c_{ij} x_{ij}^{mt} \tag{25}$$

The experiments are performed on all instances with 20 retailers and 3 periods, which are more time-efficient compared to instances with more retailers and more representative than instances with 10 retailers. As vehicles' travel speed is not a decision variable in the traditional objective, they can travel at any speed as long as they respect the speed bounds. In this situation, we may obtain various values of speed for each road, resulting in different driver wage, fuel consumption and emissions, which makes it difficult to compare the cost components of the two models. Therefore, for the traditional model, we fix vehicles' travel speed to the optimal one got from the comprehensive model, and compute corresponding cost components. Results are reported in Table 4, and Table 5 presents the two objectives' deviations.

From Table 4 and Table 5, it is observed that the traditional objective will lead to a poor total cost performance, increases by 6.71% on average, which is mainly caused by higher transportation and emission costs. Further, it consumes more fuel and emits more CO_2 (increase by 23.09% on average). It should be noted that we preset the optimal speed for the traditional model; otherwise, it will produce solutions with even more cost and emissions. Therefore, in our experimental setup the proposed comprehensive model can not only save cost, but also have a better environmental benefit.

Table 4 The difference between a comprehensive objective and a traditional objective

Instance	Traditional objective						Comprehensive objective					
	IC	DC	VFC	FC	EC	TC	IC	DC	VFC	FC	EC	TC
20-1-1-3-1	313.73	66.30	93.92	103.99	93.25	671.19	313.73	66.30	59.90	80.95	72.58	593.45
20-1-1-3-2	331.55	61.65	93.92	96.91	86.90	670.94	331.55	61.65	59.90	73.35	65.77	592.23
20-1-1-3-3	231.96	54.19	93.92	88.26	79.14	547.47	232.10	58.91	101.58	68.36	61.30	522.25
20-1-1-3-4	268.55	58.12	93.92	94.03	84.32	598.94	252.53	69.98	101.58	80.65	72.32	577.06
20-1-1-3-5	216.41	78.28	93.92	125.51	112.54	626.66	225.66	89.09	101.58	103.23	92.56	612.12
20-1-3-3-1	264.20	63.10	93.92	100.17	89.81	611.19	250.49	71.07	101.58	83.32	74.71	581.18
20-1-3-3-2	216.79	75.20	93.92	133.88	120.04	639.83	227.27	87.67	101.58	98.50	88.32	603.33
20-1-3-3-3	271.01	58.40	93.92	91.52	82.06	596.92	252.41	68.39	101.58	80.71	72.37	575.47
20-1-3-3-4	288.65	55.21	59.90	65.42	58.66	527.85	288.65	55.21	59.90	64.82	58.12	526.71
20-1-3-3-5	228.21	61.88	93.92	96.91	86.90	567.82	228.21	61.88	59.90	75.45	67.65	493.10
20-1-5-3-1	248.85	53.28	93.92	88.48	79.34	563.86	253.19	53.28	93.92	85.94	77.06	563.39
20-1-5-3-2	298.16	57.27	93.92	92.67	83.09	625.11	270.11	76.30	101.58	86.51	77.57	612.05
20-1-5-3-3	263.18	64.36	93.92	112.62	100.98	635.07	260.42	74.25	101.58	85.81	76.94	599.01
20-1-5-3-4	278.81	56.92	93.92	87.76	78.69	596.10	278.81	56.92	59.90	67.87	60.85	524.35
20-1-5-3-5	273.14	71.17	93.92	145.90	130.82	714.95	272.57	83.79	101.58	97.92	87.80	643.66

IC: Inventory holding cost; **DC**: driver cost; **VFC**: vehicle fixed cost; **FC**: fuel cost; **EC**: emission cost; **TC**: total cost

The result of instance 20-1-1-3-4 is given in Figure 3, to show the difference in terms of solutions' construction in detail, where the number in the circle is the depot or retailer, and the number over each retailer is the product quantity delivered in corresponding period, and the number on the arrow is vehicle's speed between two vertices. It shows that the traditional model tends to use heavy duty vehicles, thereby reducing travel distance, although its inventory cost will increase. As to the comprehensive model, light and medium duty vehicles are preferred. To conclude, solutions obtained from the comprehensive model differ from those generated by the traditional model in terms of inventory strategy and vehicle scheduling.

Table 5 The traditional objective's percentage deviation from the comprehensive objective

Instance	IC	DC	VFC	FC	EC	TC
20-1-1-3-1	0.00	0.00	56.79	28.47	28.47	13.10
20-1-1-3-2	0.00	0.00	56.79	32.12	32.12	13.29
20-1-1-3-3	-0.06[a]	-8.01	-7.54	29.11	29.11	4.83
20-1-1-3-4	6.35	-16.95	-7.54	16.59	16.59	3.79
20-1-1-3-5	-4.10	-12.13	-7.54	21.58	21.58	2.38
20-1-3-3-1	5.47	-11.22	-7.54	20.21	20.21	5.17
20-1-3-3-2	-4.61	-14.21	-7.54	35.92	35.92	6.05
20-1-3-3-3	7.37	-14.62	-7.54	13.39	13.39	3.73
20-1-3-3-4	0.00	0.00	0.00	0.93	0.93	0.22
20-1-3-3-5	0.00	0.00	56.79	28.45	28.45	15.15
20-1-5-3-1	-1.72	0.00	0.00	2.95	2.95	0.08
20-1-5-3-2	10.38	-24.94	-7.54	7.12	7.12	2.13
20-1-5-3-3	1.06	-13.32	-7.54	31.25	31.25	6.02
20-1-5-3-4	0.00	0.00	56.79	29.31	29.31	13.68
20-1-5-3-5	0.21	-15.06	-7.54	49.00	49.00	11.08
Average	1.36	-8.70	10.62	23.09	23.09	6.71

[a] Negative values represent that the corresponding cost in the traditional model is lower

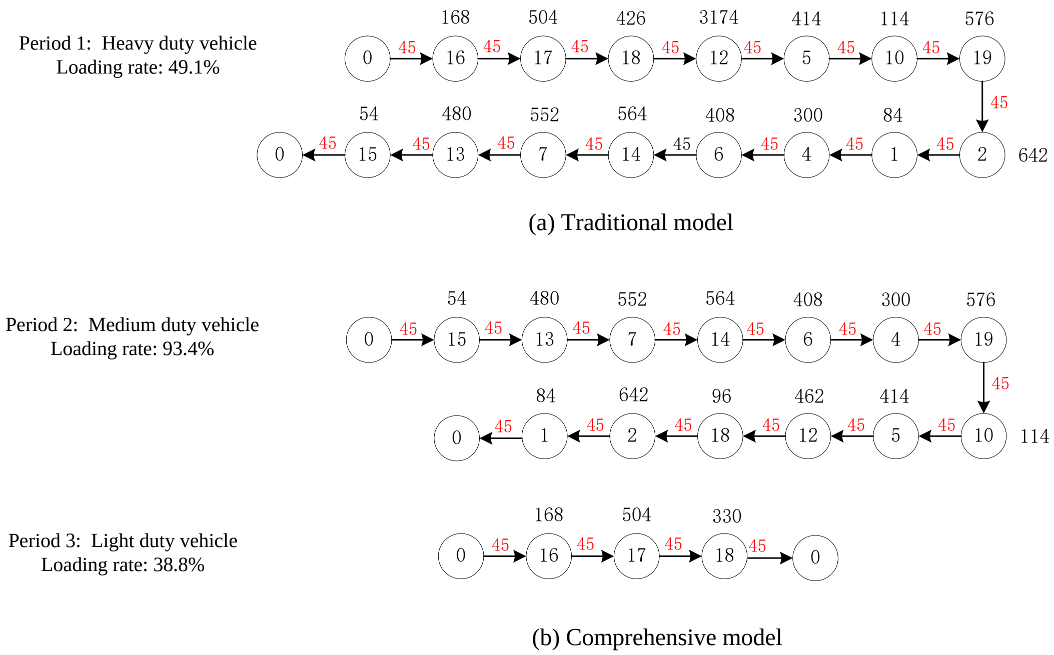

Figure 3 Solution constructions of different objectives for instance 20-1-1-3-4

5.4 Impact of a Heterogeneous Fleet

This section analyzes the influence of applying a heterogeneous fleet. To this end, we conduct four groups of experiments on instances with 10 and 20 retailers, and the time horizon is 3. The four groups of experiments include using a heterogeneous fleet and a unique vehicle type (i.e., only light duty vehicles, only medium duty vehicles and only heavy duty vehicles). Results are reported in Table 6 and Table 7. In Table 6, the “Gap” means the percentage increase in terms of cost when using a unique vehicle type as opposed to using a heterogeneous fleet. In Table 7, the gap refers to the average percentage decrease in terms of emissions when a heterogeneous fleet is used.

Table 6 demonstrates that the total cost will increase when a homogeneous fleet is used. Compared to the scenario with a heterogeneous fleet, the average increase for light duty vehicles ranges from 8.89% to 11.16%. For medium duty vehicles, the average increase is between 2.09% and 2.44%. With heavy duty vehicles, the average increase changes from 6.30% to 10.29%. These results suggest that in our experimental setup if a homogenous fleet is used, it is desirable to use the medium duty vehicles. Table 7 indicates that a heterogeneous fleet also generates low-carbon solutions. Compared with the scenario using only heavy duty vehicles, it can reduce CO_2 emissions by 18.89% and 16.00% on average for instances with 10 and 20 retailers, respectively.

Table 6 The cost benefits of using a heterogeneous fleet

Instance	Heterogeneous fleet	Only light duty	Only medium duty	Only heavy duty	Instance	Heterogeneous fleet	Only light duty	Only medium duty	Only heavy duty
	Total cost	Gap (%)	Gap (%)	Gap (%)		Total cost	Gap (%)	Gap (%)	Gap (%)
10-1-1-3-1	442.97	8.14	0.00	15.05	20-1-1-3-1	593.45	8.69	0.00	13.08
10-1-1-3-2	402.01	0.00	6.51	23.61	20-1-1-3-2	592.23	4.24	0.00	12.65
10-1-1-3-3	401.48	4.04	0.00	13.84	20-1-1-3-3	522.25	16.82	3.66	3.87
10-1-1-3-4	481.31	10.75	0.00	13.15	20-1-1-3-4	577.06	9.67	3.61	3.13
10-1-1-3-5	534.93	11.83	0.00	13.65	20-1-1-3-5	612.12	12.09	3.61	2.21
10-1-3-3-1	511.45	10.85	4.16	3.45	20-1-3-3-1	581.18	8.10	3.47	5.17
10-1-3-3-2	455.26	17.64	4.17	3.70	20-1-3-3-2	603.33	14.23	3.71	1.26
10-1-3-3-3	509.16	9.68	4.11	1.43	20-1-3-3-3	575.47	11.73	3.46	3.73
10-1-3-3-4	475.25	8.56	4.44	2.74	20-1-3-3-4	526.71	11.17	0.00	13.44
10-1-3-3-5	411.53	6.57	0.00	16.16	20-1-3-3-5	493.10	13.05	0.00	15.15
10-1-5-3-1	435.40	12.50	0.00	15.34	20-1-5-3-1	563.39	14.27	4.85	0.00
10-1-5-3-2	434.01	2.47	0.00	15.63	20-1-5-3-2	612.05	10.51	3.71	2.13
10-1-5-3-3	522.65	10.11	3.68	0.72	20-1-5-3-3	599.01	9.81	3.34	3.36
10-1-5-3-4	461.95	4.70	0.00	14.84	20-1-5-3-4	524.35	10.38	0.00	13.68
10-1-5-3-5	502.87	15.56	4.23	1.10	20-1-5-3-5	643.66	12.63	3.18	1.67
Average	465.48	8.89	2.09	10.29	***Average***	574.62	11.16	2.44	6.30

Table 7 The environmental benefits of using a heterogeneous fleet

Instance	Emission gap (%, Only light duty)	Emission gap (%, Only medium duty)	Emission gap (%, Only heavy duty)
All instances with 10 retailers	-8.56	-0.87	-18.89
All instances with 20 retailers	-13.55	-1.04	-16.00

Figure 4 displays the solution construction for instance 10-1-5-3-5 under the four scenarios. First, results differ from each other in terms of vehicles' capacity utilization, which reaches its maximum level when using a heterogeneous fleet and only light duty vehicles. The loading rate for a heavy duty vehicle is the lowest.

Second, vehicle schedules are different. Take Figure 4.(a) and 4.(b) for example, the visiting time are changed for retailers 3, 7 and 9. When using a heterogeneous fleet, these three retailers are visited in periods 2 and 3; however, when only light duty vehicles are available, they are visited in every period. Moreover, inventory strategies are distinct, which is obvious when retailers' visiting time is changed.

From Figure 4, it is noticed that vehicles travel at a constant speed, even if the payload is decreasing along the trip due to unload at each retailer. Although vehicles can travel faster when becoming lighter, thereby reducing driver cost; however, as speed increases, vehicles might consume more energy and emit more CO_2. Thus, it is more economical for them to travel at a constant speed under our experimental setup. Note that this phenomenon is just a coincidence in our study, and it does not mean that in all situations vehicles must travel at a constant speed.

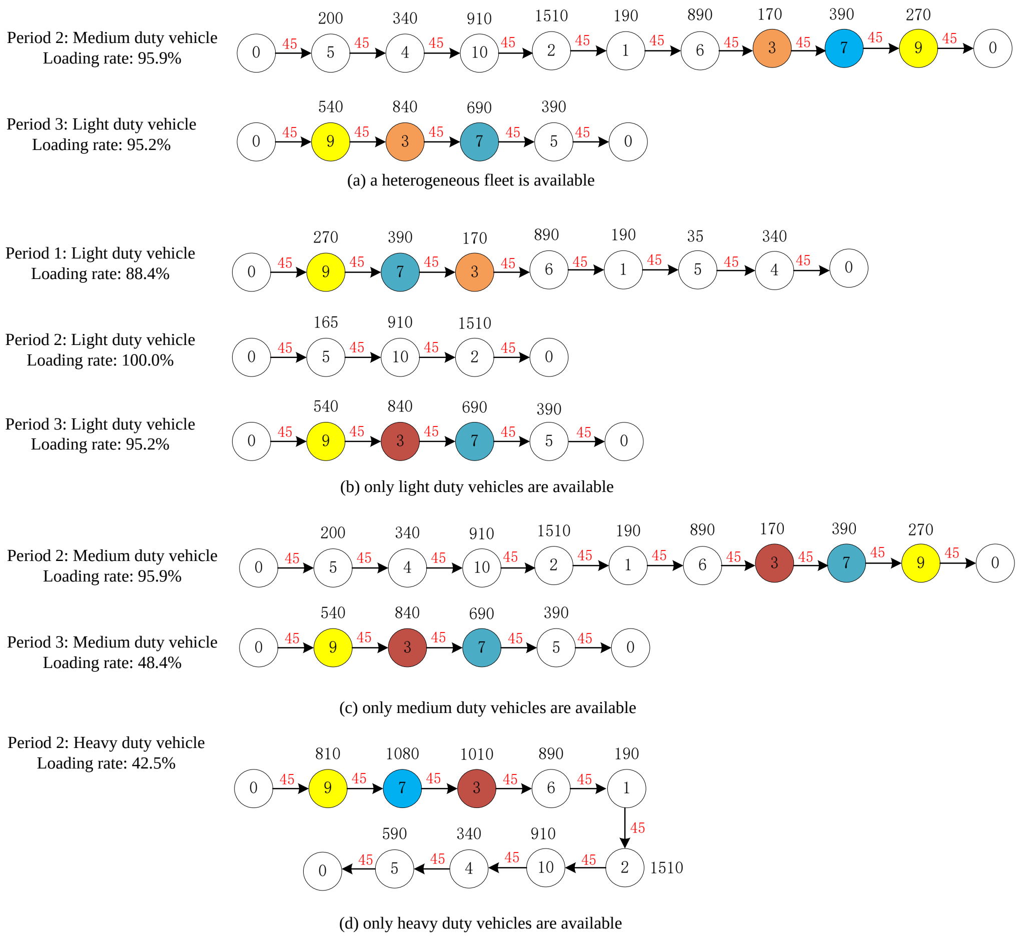

Figure 4 Solution constructions for instance 10-1-5-3-5 when vehicle types are different

5.5 Parameter Analyses

This section analyzes the effects of parameters on cost and CO_2 emissions. All our tests are performed on instance 20-1-1-3-2, which is more time-efficient in terms of CPU.

5.5.1 Unit inventory holding cost

From the objective function, we find that inventory holding cost coefficient (h_i) will influence system decision. Since in the instance data the value of each h_i is different, we change all vertices' unit inventory holding cost through multiplying by a coefficient α.

Figure 5 shows the trends of cost and emissions with changing value of h_i. The total cost increases almost linearly over h_i. However, the emission curve demonstrates a "staircase pattern". That is, under some circumstances, the system's emission level keeps constant while the total cost increases with higher h_i values. It can be concluded that to achieve the same emission level, the cost may vary significantly for different industries, and that it is difficult for companies with higher inventory cost to control emissions at a lower price. Thus, government should consider the difference of industries when implementing emission legislations.

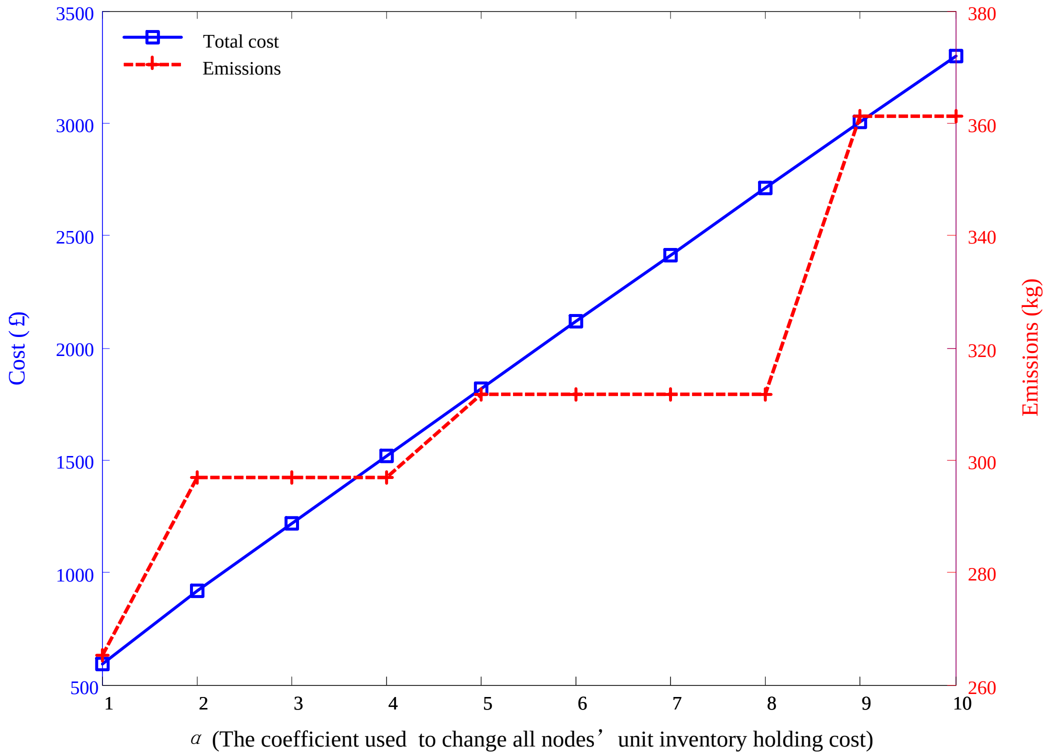

Figure 5 The trend of costs and emissions when h_i changes

In Figure 5, there are four break points in terms of CO_2 emissions, i.e., α =1, 2, 5 and 9. Figure 6 presents vehicles' routes and product quantities delivered in these four scenarios, to demonstrate the detailed impacts. First, it shows that h_i will influence delivery frequency. When h_i is higher, the system tends to delivery fewer products in period 2, aiming to lower inventory; which on the other hand will cause more trips and longer travel distance, leading to more emissions (refer to Figure 6.(a) and 6.(b)).

Further, h_i will affect the number of retailers visited in each trip. Take Figure 6.(b), 6.(c) and 6.(d) for examples, although in these subgraphs the vehicle type used in period 2 is the same, the number of retailers visited decreases.

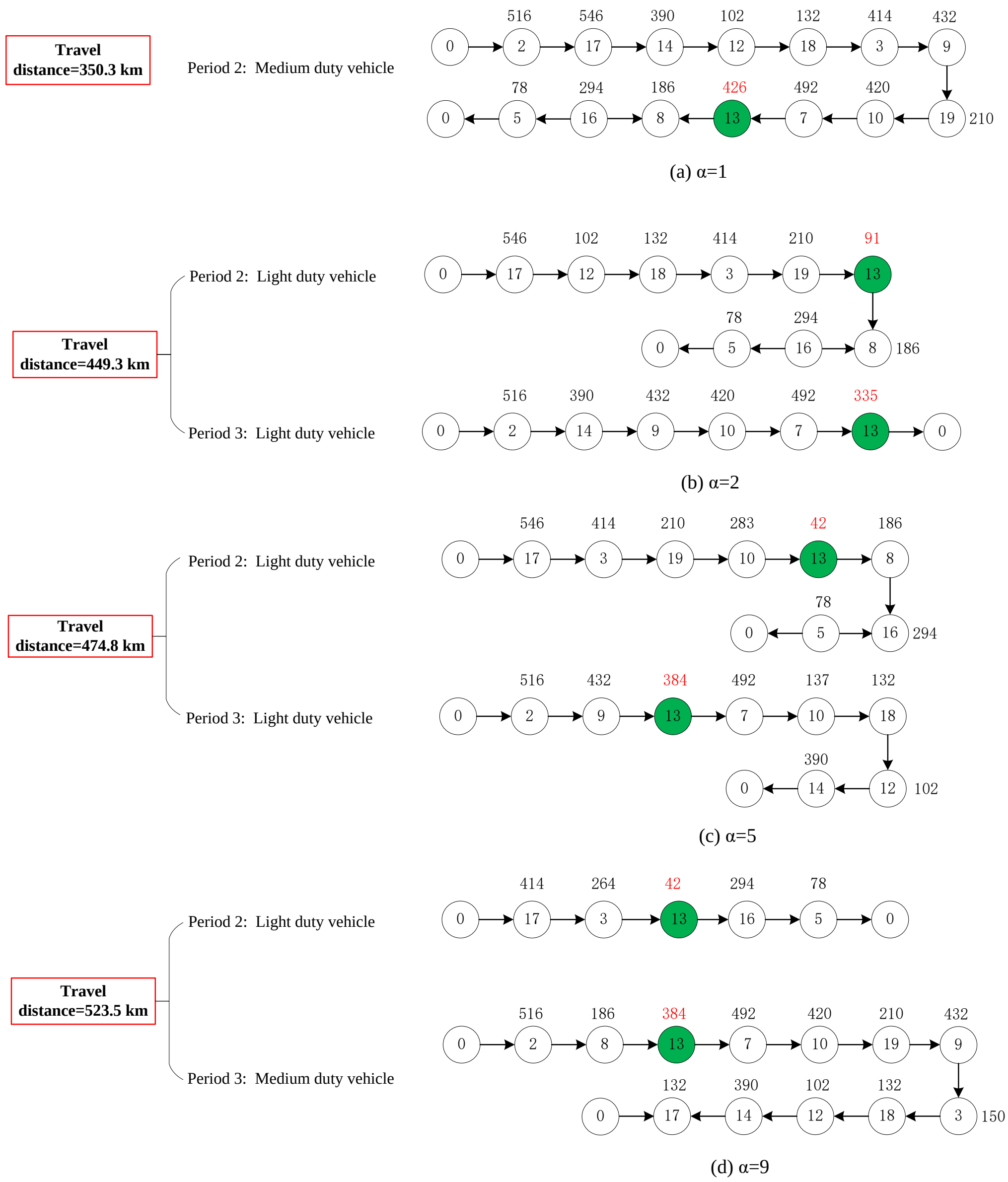

Figure 6 The effect of unit inventory holding cost on vehicle scheduling and inventory strategy

5.5.2 Unit emission price

Figure 7 suggests the influence of unit emission price on the system's cost and CO_2 emissions. It is found that the total cost increases almost linearly with the increase of emission price and that on the whole the emission curve exhibits a decreasing trend. When emissions are not considered (i.e., unit emission price = 0), the system emits the most CO_2 as expected. However, a higher carbon price (or taxing) does not always lead to a better environment benefit. For example, when the price increases from 0.496 *£/kg* to 1.984 *£/kg*, the emission level does not change, but the system's cost increases dramatically. Therefore, when implementing carbon regulation policies, governments should carefully determine the carbon price, under which enterprises can reduce emissions without heavy burdens on costs. In addition, since fuel consumption and emissions are linearly dependent, Figure 7 can also represent the tendencies of cost and emissions under changing fuel price.

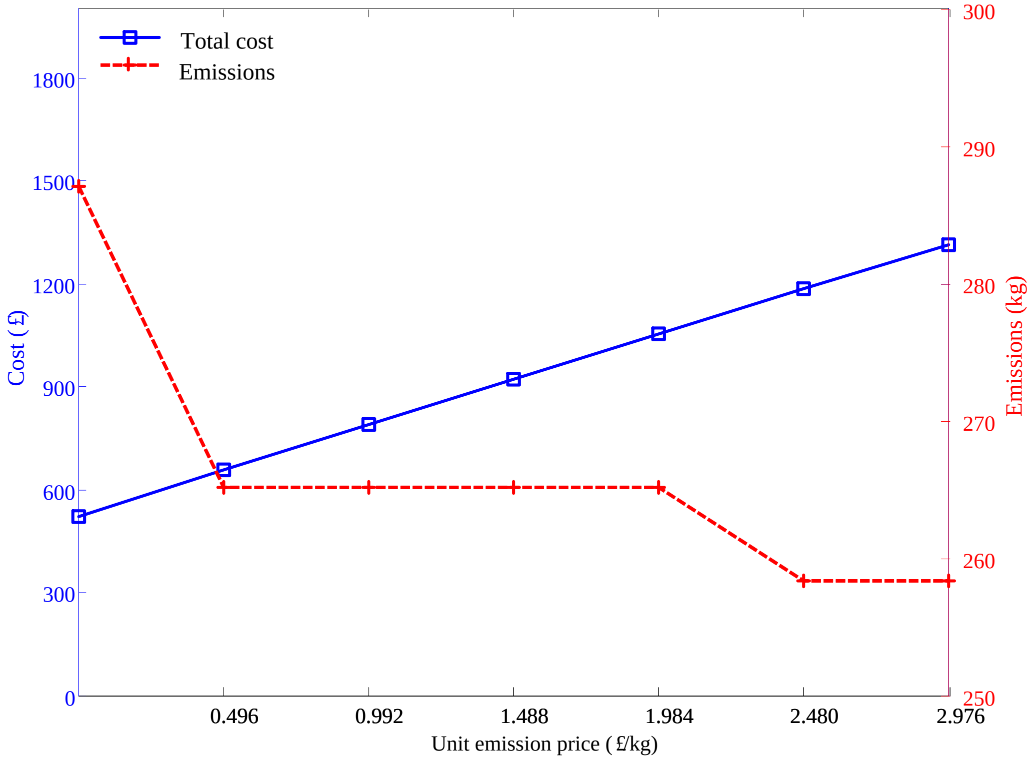

Figure 7 The effect of unit emission price on cost and emissions

In Figure 7, there are three break points in terms of CO_2 emissions, i.e., f_e =0, 0.496, and 2.480. Figure 8 presents the detailed constructions of solutions in these three scenarios. It shows that when emission cost is not considered, the vehicle tends to travel faster, in order to reduce driver cost, which leads to more CO_2 emissions. As f_e increases to 0.496, vehicles start to travel at a lower speed, aiming to reduce fuel consumption and emissions. However, the system's decision keeps unchanged when f_e varies from 0.496 to 1.984. In these cases, if the decision makers choose to further lower travel speed, the driver cost will increase, more than the decrease of fuel and emission costs. When f_e =2.480, vehicle's speed is reduced to 35 *km/h*.

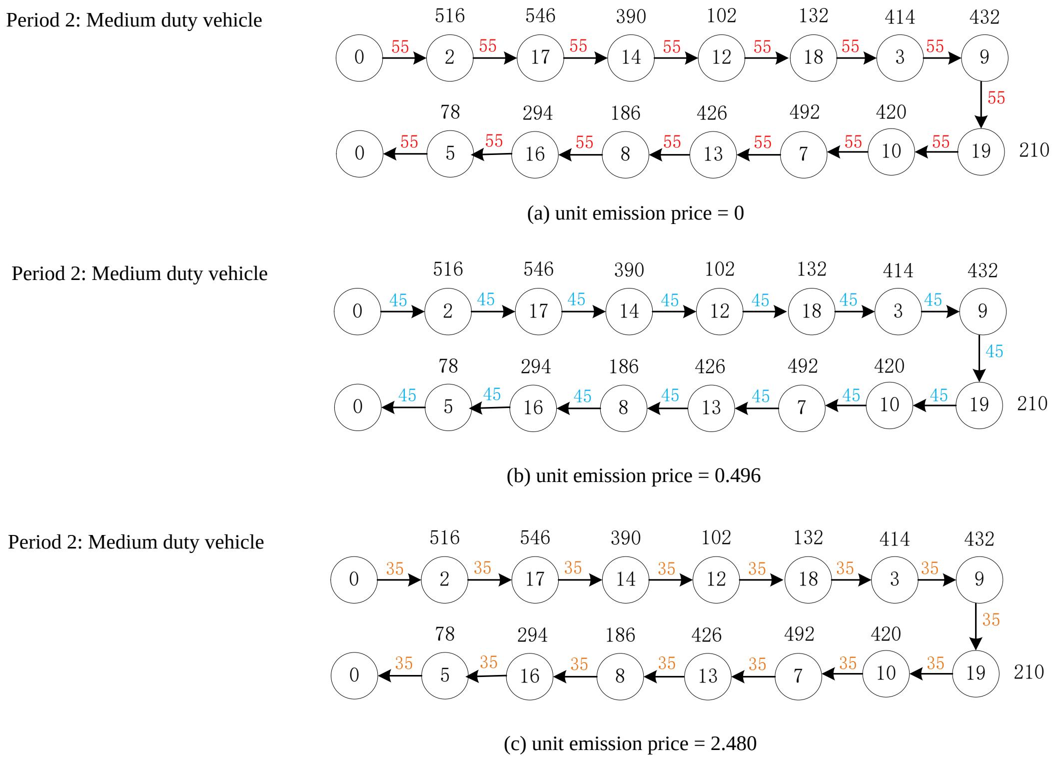

Figure 8 The effect of unit emission price on vehicle scheduling and inventory strategy

6. CONCLUSION and FUTURE WORK

This paper studies an inventory routing problem simultaneously considering environmental issues and a heterogeneous fleet, where fuel consumption and emissions are influenced by load, distance, speed and vehicle characteristics. A comprehensive objective comprising inventory cost, driver wage, fixed transportation cost, fuel and emission costs is proposed. Numerical tests quantify the benefits of using a comprehensive objective and a heterogeneous fleet, including saving costs and reducing CO_2 emissions. Parameter analyses display that it is difficult for companies with high inventory cost to control emissions at a lower price. Moreover, we find that a higher carbon (or fuel) price does not always mean a better environmental benefit, which can provide suggestions to governments when implementing emission regulation policies.

Future study can be conducted in four aspects: (1) our results demonstrate that vehicles tend to travel at a constant speed during a trip instead of changing speed on each arc. Since Bektaş and Laporte (2011) suggest that the impact of speed is more obvious when time windows are imposed, future research can include time window constraint and further investigate the influence of speed; (2) including the fuel consumption and emissions in inventory activities, especially for the products which require being stored in low-temperature environment; (3) developing a multi-objective optimization model to capture the tradeoff between costs and emissions; (4) developing powerful heuristic algorithms to solve the model presented here, which is able to handle large-sized instances.

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