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Flow-based Integer Linear Programs to Solve the Weekly Log-Truck Scheduling Problem

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Abstract In this paper we present the solution to a weekly log-truck scheduling problem (LTSP) integrating the routing and scheduling of trucks where all goods are transported in full truckloads. This problem includes aspects such as pick-up and delivery, multiple products, inventory levels, and lunch breaks. The objective is to minimize the overall transportation cost including wait times, and both empty and loaded distance traveled. Our solution is based on a two-phase approach. The first phase involves an integer linear program that determines the destinations of full truckloads. The second phase uses an implicit integer linear program based on an arc formulation model, ensuring routing and scheduling of trucks at a minimum cost. Experiments have been conducted using Cplex 12.4.0, and almost all instances were solved within six hours with a reasonable gap.

Keywords Forestry · Transportation · Routing · Scheduling · Mixed-integer programming

1 Introduction

The forest industry manufactures goods from timber grown in forests. It provides a large range of products such as paper, wrappings, building mate-

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rials and furniture. As wood harvesting has advanced over the years, public concerns and ancillary uses of the forest have developed in parallel with the development of the forest industry (recreation, and the preservation of wildlife species and vegetation). The forest industry occupies an important place in the economy of several countries such as Chile, Canada, Sweden, Finland, New Zealand and Austria. **In Canada, the industry employed about 600,000 persons in 2011, including 233,900 directly. In the same year, it contributed $23.7 billion to Canada's economy (about 1.9% of Canada's GDP) [11]. Forests cover a large part of Sweden and Finland, allowing these countries to develop a flourishing forest industry that contributes about 8% to Finland's GDP, putting it in second place after the electronic sector [12].**

Forest operations consist of many different activities such as planting, building the routes to access forest locations and transporting the harvest to the wood mills. In Canada, the distances between forest locations and wood mills are quite large. Furthermore, backhauling, which is most of the time performed empty, represents a waste of resources (time, fuel, etc.). It is thus important to reduce unproductive activities during transportation, both for economic and environmental considerations.

In this paper, we consider a variant of the weekly log-truck scheduling problem (LTSP), which consists in assigning trucks to move wood loads from forest locations to paper mills. In this variant, trucks and log-loaders need to be synchronized for loading and unloading operations. The purpose of this paper is to address a number of new challenges that were not considered in El Hachemi *et al.* [6], such as lunch breaks, supply constraints, and home bases of trucks.

The paper is organized as follows. Sections 2, 3 and 4 present respectively the LTSP, the literature review and the approach adopted to solve the problem. The experimental setting is described in Section 5, where computational results are also reported. Section 6 concludes the paper.

2 Log-Truck Scheduling Problem

In this paper, we consider a set F of forest areas, a set M of wood mills, a set P of wood assortments and a planning horizon corresponding to a set J of days. On day j of the planning horizon, wood mill $m \in M$ requires a (demand) quantity D^j_{mp} of assortment $p \in P$. Similarly, a weekly (supply) quantity S_{fp} of product $p \in P$ is available in forest area $f \in F$. Demand and supply quantities are expressed in number of full truckloads. Loads are transported from forest areas to wood mills by trucks, one load at the time. In each forest location and at each wood mill, a single log-loader is available for loading/unloading operations. Therefore, a single truck can be

loaded/unloaded at any time in any location. If several trucks are present, these trucks, except for one, must wait for the log-loader to become available. These waiting times significantly increase transportation costs, since in most of the times, the drivers leave the engine running while they are waiting to load or unload, this situation must be avoided as much as possible. It should be noted that although less-than-truckload operations are common in other countries, they are seldom in Canada as the harvested volumes are quite large. Since full truckloads is the main mode of operation, we choose to express all quantities in truck loads rather than in cubic meters.

The LTSP consists in constructing a transportation plan over the planning horizon in which the required number of loads is delivered to wood mills at minimum total cost while respecting constraints on supplies in forest areas and stock constraints at wood mills. The objective function includes both the cost of unproductive activities (waiting times and deadheads) and the transportation cost of full truckloads from forest areas to wood mills. In our application, we also consider that we have a set of regional bases. Each base is associated with a set of trucks, so that each truck must begin and end each day at its base. In this paper, we consider instances with one and three bases.

The LTSP is related to routing problems encountered in other industries, in particular, the *Vehicle routing problem (VRP)* and *Pick-up and delivery problems with time windows (PDPTW)* (see the book edited by Toth and Vigo [21] where variants of the problem and different algorithms are discussed), but it differs significantly from these problems.

3 Literature Review

Since the mid-1990s, several projects have been conducted in the forestry sector aiming at improving the efficiency of the transportation activities such as the control and quality of truck scheduling. Among these, one should note the seminal work of Weintraub *et al.* [22] in Chile in the ASICAM project, as well as the EPO system developed by Linnainmaa *et al.* [13] in Finland. More recently, Palmgren *et al.* [15], [16] have proposed a column generation scheme for tackling the LTSP. More details on these works can be found in El Hachemi *et al.* [5].

Tabu Search (TS) heuristics were also applied by Gronalt and Hirsch [10] to solve a restricted variant of the LTSP, where the destination of each load is given *a priori*. El Hachemi *et al.* [5] considered the same problem and proposed a two-step hybrid solution procedure for it: in the first step an assignment model produces a set of deadhead movements while a Constraint Programming (CP) model is used in the second step to schedule activities taking into account

the synchronization constraints between trucks and log-loaders. In [6], the same authors revisit this problem and propose a constraint-based local search procedure for solving it. This procedure is coupled with a mixed integer programming model to address the weekly LTSP, in which the assignments of loads from forest areas to mills must be optimized.

Flisberg *et al.* [9] and Andersson *et al.* [1] proposed a two-phase approach to deal with the daily problem. A first LP model is used to determine the flow of wood from supply points to demand points, which is then passed to a second model which sequences these transportation nodes into complete routes by using a standard tabu search. The approach is run in real time, as a dispatching procedure continuously monitors the truck routes during the day and performs the necessary updates. This procedure, which builds a route a single trip at a time, is inspired from Rönnqvist and Ryan [20] and Rönnqvist *et al.* [19]. Rey *et al.* [17] addressed the problem of scheduling the daily assignment of available trucks for the delivery of forest products required at different destinations. In this study, they proposed an integer programming model based on a column generation formulation, each column representing a given truck's trip schedule for a working day. The solution approach is based in first finding the optimal solution of the linear relaxation of the model, and then solving the integer model constructed with all the columns generated previously. **The approach proposed in this paper differs mainly from these papers with regards to the synchronization of trucks and loaders, which they do not consider. For a comprehensive discussion of synchronization problems in several sectors, see Bredström and Rönnqvist [3], Eveborn *et al.* [7] and Eveborn *et al.* [8].**

With respect to problem size, most papers (Murphy [14], Palmgren *et al.* [15], Gronalt and Hirsch [10], El Hachemi *et al.* [5]) present cases studies which range from 9 to 28 trucks and 30 to 85 transport tasks per day. Flisberg *et al.* [9] and Andersson *et al.* [1], on the other hand, have solved substantially larger problems ranging from 188 transport tasks to about 2,500 full truckloads with 15 to 110 trucks. In this paper we solve the weekly problems of El Hachemi *et al.* [6] having up to approximately 700 transport tasks and around 14 to 32 trucks and introduce to new instances of about the same size. For a more detailed description of optimization problems in the forest sector, we refer the reader to Rönnqvist [18] and D'Amours *et al.* [4].

4 Solution Approach

In this paper, we focus on developing a solution method to solve a modified version of the weekly LTSP allowing the wood mills to operate in just-in-time mode. El Hachemi et *et al.* [6] propose a hybrid local search (LS)/CP method to deal with this problem. However, new challenges, such as accounting for lunch breaks, supply constraints and the home base of trucks, led us to look for a new model that better represents the situation at hand.

As in El Hachemi *et al.* [6], a two-phase approach is used in this paper to solve the weekly LTSP. The first phase, which we call the "tactical problem", **is identical to the one presented in El Hachemi *et al.* [6];** it involves solving an IP formulation taking into account demand, supply and stock constraints. In this step, restrictions are on time availability of trucks. This phase yields seven daily LTSPs whose solution make up the second phase. For each of these LTSPs, there is a fixed set of transportation requests to perform. **The novelty of this paper consists in proposing** a new flow-based IP model to address all the other issues of the daily LTSP. Each component of a truck trip is represented in the network as an arc, while constraints are enforced through capacity constraints on truck flows. Since the tactical model has been already presented in El Hachemi *et al.* [6], we briefly recall the tactical problem before presenting the flow-based model of the second phase of our approach.

4.1 The Tactical Problem

In the tactical model, we take into consideration the fact that different wood products must be shipped to wood mills. These multi-product demand constraints arise from the fact that many wood mills order logs in specific lengths and diameters to produce given final products, as well as the fact that the properties of wood are strongly related to the particular tree species that yield it. Thus, the logs are sorted into different assortments that depend on species, usage, quality and dimension. Each supply point consists of a given assortment group (up to 5 products in our case) and each demand point represents a requirement of a given assortment group. In general, the inventory is known at the beginning of the week since it is the stock associated with the last day of the previous week. In some cases, when the demands at wood mills and supplies at forest areas remain constant during a long period covering many weeks, it is judicious to generate a weekly solution that can be repeated during the whole period. To achieve this goal, we impose that the inventory at the beginning of the first day is equal to the inventory at the end of the last day of the week.

In this phase, the objective function is to minimize the cost of full truckloads subject to the following contraints. First, we ensure that the daily stock of any product at any wood mill does not exceed a given capacity. Second, we satisfy the daily demands of each product of wood mills over the whole week. We consider an upper limit on the number of hours that a loader may operate on any day. Similarly, if a loader is active on any given day it must operate for a minimum number of hours. These limits can also be expressed in loads.

4.2 Phase 2: The Daily Synchronized Log-Truck Scheduling Problem

We present an IP model to deal with the daily synchronized log-truck scheduling problem (SLTSP). This model is very close to a network flow formulation. Each component (activity) of a truck trip (deadhead, loading, loaded travel, unloading) is modeled as an arc in a network, as well as truck waiting times (see Figure 1). Our model relies on a space-time representation of truck movements. Time is discretized in time slices of a fixed given duration. The time step is a key parameter of the model, since the precision of solutions is closely related to it.

4.2.1 Parameters

B	:	the set of all bases,
c^l	:	the cost of waiting time of a log-loader per unit of time,
c_a	:	the cost associated with arc a (empty driven arcs and trucks waiting times),
t^l	:	the loading time of one shipment,
t^u	:	the unloading time of one shipment,
H	:	the optimization horizon,
T_a^s	:	the starting time associated with arc a,
T_a^e	:	the ending time associated with arc a,
K_{fm}	:	the number of full truckloads to be delivered between forest area f and wood mill m,
L_{bfm}	:	the set of loaded trip arcs linking forest area f and wood mill m and associated with base b,
L_b	:	$\cup_{f \in F} \cup_{m \in M} L_{bfm}$ the set of loaded trip arcs associated with base b,
$A^+(s_b)$	:	the set of exiting arcs from the source node s_b associated with base b,
$A^-(t_b)$	:	the set of entering arcs into the sink node t_b associated with base b,
$A^+(i)$	:	the set of exiting arcs from the node i,
$A^-(i)$	:	the set of entering arcs into the node i,
A_{bf}^T	:	the set of loading arcs (a_b) loading at forest area f and associated with base b, such that their starting times $T_{a_b}^s \in \left[T, T + t^l\right[$, $T \leq H$,
A_{bm}^T	:	the set of unloading arcs (a_b) unloading at wood mill m and associated with base b, such that their starting times $T_{a_b}^s \in [T, T + t^u[$. $T \leq H$,
C_{bf}	:	the set of loading arcs associated with forest area f and base b,
C_b	:	$\cup_{f \in F} C_{bf}$ the set of loading arcs associated with base b,
D_b	:	the set of unloading arcs associated with base b,
E_b	:	the set of empty driven arcs associated with base b,
W_b	:	the set of trucks waiting arcs associated with base b,
N_b	:	the set of nodes associated with base b,
V_b	:	the number of trucks of base b.
A	:	the set of all arcs.

4.2.2 Variables

x_{a_b}	:	the number of trucks that follow arc $a_b \in A$.
t_f^{min}	:	the start time of log-loader associated with forest area f.
t_f^{max}	:	the end time of log-loader associated with forest area f.

4.2.3 Flow based model

$$\text{Min} \sum_{b \in B} \sum_{a_b \in E_b \cup W_b} c_{a_b} x_{a_b} + \sum_{f \in F} c^l (t_f^{max} - t_f^{min})$$

$$\sum_{a_b \in A^+(s_b)} x_{a_b} \leq V_b, \ \forall b \in B \quad (1)$$

$$\sum_{a_b \in A^+(s_b)} x_{a_b} = \sum_{a_b \in A^-(t_b)} x_{a_b}, \ \forall b \in B \quad (2)$$

$$\sum_{a_b \in A^+(i_b)} x_{a_b} = \sum_{a_b \in A^-(i_b)} x_{a_b}, \ \forall b \in B, \ \forall i_b \in N_b - \{s_b, t_b\} \quad (3)$$

$$\sum_{b \in B} \sum_{a_b \in A_{bf}^T} x_{a_b} \leq 1, \ \forall f \in F, \ \forall T \leq H \quad (4)$$

$$\sum_{b \in B} \sum_{a_b \in A_{bm}^T} x_{a_b} \leq 1, \ \forall m \in M, \ \forall T \leq H \quad (5)$$

$$(H - T_{a_b}^s) x_{a_b} \leq H - t_f^{min}, \ \forall b \in B, \ \forall f \in F, \ \forall a_b \in C_{bf} \quad (6)$$

$$T_{a_b}^e x_{a_b} \leq t_f^{max}, \ \forall b \in B, \ \forall f \in F, \ \forall a_b \in C_{bf} \quad (7)$$

$$\sum_{b \in B} \sum_{a_b \in L_{bfm}} x_{a_b} = K_{fm}, \ \forall f \in F, \forall m \in M \quad (8)$$

$$x_{a_b} \in \{0, 1\}, \ \forall b \in B, \ \forall a_b \in C_b \cup D_b \cup E_b \cup L_b \quad (9)$$

$$x_{a_b} \in \{0, 1, 2, ..., V_b\}, \ \forall b \in B, \ \forall a_b \in W_b \quad (10)$$

The objective function is to minimize the daily deadheads and waiting time cost of trucks and forest log-loaders. Constraints (1) allow to respect the available fleet of vehicles related to each regional base. Constraints (2) and (3) are flow conservation constraints related to each base. Constraints (4) and (5) ensure that each loader is serving only one truck at a given time. Constraints (6) and (7) compute the starting time and ending time of each forest log-loader. **It is important to note that computing the starting time and ending time of each forest log-loader is also associated with the objective function. Since, it attempts to minimize the overall cost of unproductive activities, the optimizer will seek to set each variable t_f^{min} to its largest possible value while satisfying constraints (6) and set each t_f^{max} to its smallest possible value while satisfying constraints (7)**. Constraints (8) indicate the daily number of requests to be

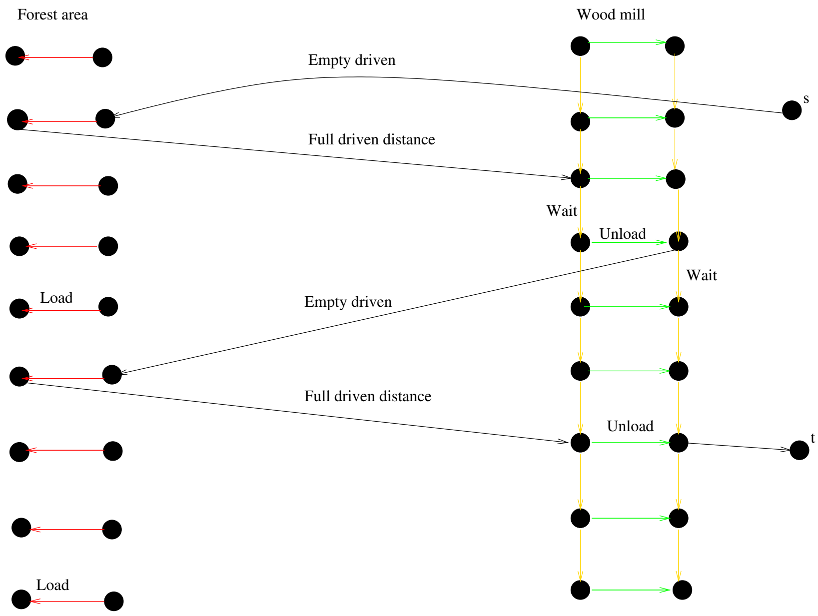

Fig. 1 Network Arcs Formulation Model

met. Constraints (9) express the fact that all arcs, except waiting time ones, have a unit capacity. Finally, constraints (10) represent the fact that for each base b, the number of trucks waiting before or after unloading cannot exceed V_b.

Forest companies need to ensure a one hour break between 11 a.m. and 3 p.m. for each truck at any wood mill, where drivers can eat lunch and trucks can be refueled. To satisfy this new constraint, we divide the network into two parts at wood mills nodes, one before the break and one after. The arcs linking both parts represent a break having a duration of one hour, as specified by the forest companies (See Figure 2). It is important to note that a truck waits before leaving the wood mill or before being loaded in a forest area. Since we expect, in general, the number of mills to be significantly smaller than the number of forest areas, we have decided to make trucks wait at mills. For that reason, waiting arcs do not appear in the representation of forest areas.

This network (See Figure 2) contains four node types: start, end, forest area and wood mill. There is a single start node (s) and single end node (t) for each base and each period of the horizon. There is a pair of nodes grouped by forest area and wood mill for each period of the horizon and each base. This clustering of nodes represents implicitly the spatial dimension of the problem. All nodes are sorted in chronological order; this is represented vertically in Figure 2. The network involves eight arc types: start of schedule (from start nodes to forest area nodes), end of schedule (from wood mill nodes to end nodes), loaded trip (from forest area nodes to wood

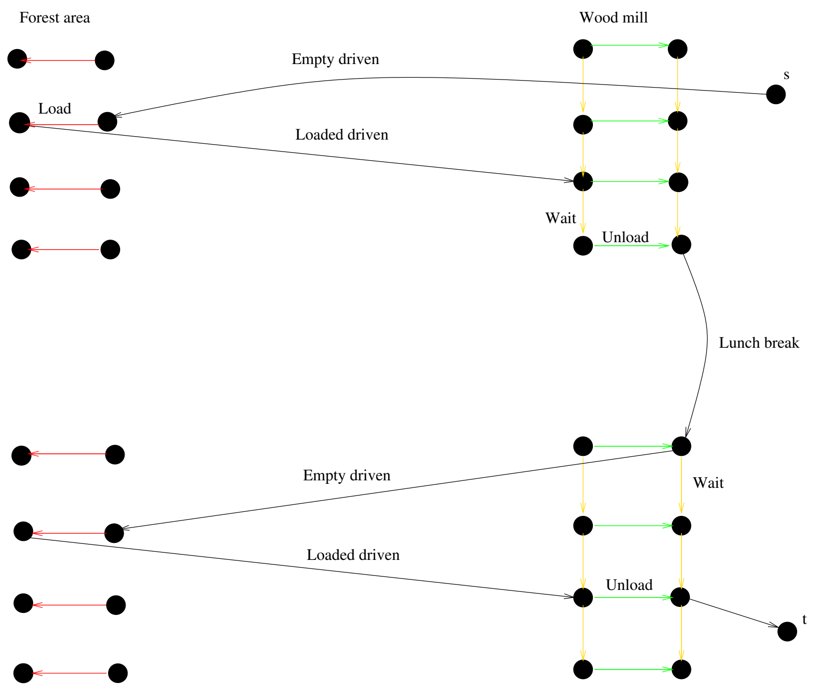

Fig. 2 Enriched Network Model

mill nodes), deadhead (from wood mill nodes to forest area nodes), break (linking both parts of the network), waiting (linking vertically two successive wood mill nodes), loading (linking horizontally two successive forest area nodes) and unloading (linking horizontally two successive wood mill nodes). For start of schedule, end of schedule, loaded trip and deadhead arcs, the cost is proportional to the associated distance and the operation cost provided by our partner. Each waiting time arc has a fixed duration and therefore a fixed cost proportional to its operation cost. It should be noted that loads are not represented in this network, however constraints (8) ensure that enough loads are transported between each forest area and each wood mill.

4.2.4 Discretization

One of the most interesting attributes of our study is that we can exploit the fact that loading and unloading times are approximatively equal (around 20 minutes, even if, in practice, loading takes a bit more than unloading). This property allows us to choose the discretization step equal to the loading time, thus it leads to a simplification of constraints and the removal of constraints (4) and (5) from the model. This is particularly advantageous in the case where we have only one regional base, since ensuring each log-loader can not serve more than one truck at the same time is automatically enforced by the flow conservation constraints, and the unit capacity on the loading arc.

4.2.5 The algorithmic approach to the synchronized log-truck scheduling problem

We propose an approach based on two parts executed sequentially to deal with each daily problem resulting from the tactical problem. Respectively, a 20-minute step-size and 10-minute step-size are used in the first part and the second part. All daily problems have been run with a fixed computational time which depends only on the number of bases involved. It must be noted that the second part is performed only if the relative gap in the first part becomes less than 1%. We have adopted a warm start strategy, since the initial solution of the 10-minute step-size part is the best one found in the 20-minute step-size part. The goal of this implementation is to use 10-minute step-size model to refine the best solution found by 20-minute step-size part.

4.2.6 Branching strategy

We developed a branching strategy that we call "Start Late End Early" (SLEE) based on reducing forest loader wait time cost in order to minimize the wait time cost component of the objective function. We focused on forest loader wait times since hourly log-loader wait time costs are approximately double the hourly cost of a truck. To address this issue, we use two loading arcs (variables) belonging to the same forest area, one in each direction, so that we branch alternatively on the earlier and then the later partial loading arcs (variables). This strategy leads to a reduction in the work time for each forest log-loader, and since the daily requests in the second phase are known in advance (a result of phase 1), this approach will induce the minimization of forest log-loader wait times. **Thus, the role of the branching strategy SLEE is to help the optimizer set each variable t_f^{min} to its largest possible value by branching on the earlier fractional x_{a_b} and to set each variable t_f^{max} to its smallest possible value by branching on the later fractional x_{a_b} such that $a_b \in \cup_{b \in B} C_{bf}$.**

5 Experimental Results

We were provided with four different case studies by FPInnovations[1]. Both case 1 and 2 involve six forest areas and five wood mills. The first case study involves approximately 400 shipments (logs) per week, and has an average cycle time of about 4 hours to transport a shipment, while the second case involves approximately 700 shipments per week with an average cycle time of about 5.5 hours to transport a shipment. Both cases have three different

[1] FPInnovation is a private, not-for-profit research and development organization whose mission is to improve Canadian forestry operations.

products to transport. **Cases 3 and 4 deal respectively with nine and eleven forest areas while the number of wood mills is equal to seven for the two cases. The number of shipments per week is repectively 560 and 583 and average cycle times are of about 6.3 and 4.16 hours. Finally, the last two cases have five products to transport. The data for these case studies can be found on the web at URL** http://w1.cirrelt.umontreal.ca/louism/Data LTSP.xls.

FPInnovations also provided approximate operations cost, \$60 per hour for each truck if it is waiting and \$70 if it is traveling. However, each loader costs more than a truck: \$100 per hour when it is waiting. Several tests have been performed by varying the number of trucks. For each of these cases, we performed two tests, the first one with only one regional base, the second one with three regional bases. **Each test consists in solving seven daily instances (weekdays) resulting from the tactical phase. Finally, all daily SLTSPs have been tested by varying the number of trucks and have been run exactly for two hours for the one-regional base case and 6 hours for the three-regional base case. In the tactical phase, we fixed the computation time to 5 minutes. All tactical problems of this paper were solved optimally in less than 1 minute. The associated results are reported in Table 1. All tests were run on a cluster of Intel Itanium II 1.5 GHz processors.**

Case	Full truckloads cost (\$)
Case 1	24,990
Case 2	80,103
Case 3	66,407
Case 4	73,733

Table 1 Tactical phase results

To assess the advantage of the method presented in this paper, we decided to compare this approach with one that has been recently published by El Hachemi *et al.* [6]. For both cases 1 and 2, we compare the best solutions associated with the ILS/CP approach to solutions developed by the approach described in this paper (see Table 2). It should be noted that $V = \sum_b V_b$. Examination of the results demonstrates clearly that the proposed (flow-based) approach outperforms the previous one (ILS/CP). We note that the difference between the approaches was considerable for the second case, where fewer trips and vehicles were involved.

We report in Table 3 and Table 4 the average gap, the maximum and minimum gap, **the average gap to the Best Known Solution (BKS) where BKS is the best solution found by all approaches proposed in this paper, the average waiting time cost, the average total cost and finally, the time to reach best solution. Comparing the Default Branching Strategy (DBS) of Cplex 12.4.0 with the SLEE strategy for the proposed flow-based formulation model shows that the SLEE**

Case	V	ILS/CP approach ($)	Flow-based approach ($)	Improvement (%)
Case1	14	54,170	52,743	2.63
	15	53,430	51,503	3.60
	16	53,043	51,590	2.73
Case2	30	137,883	130,394	5.43
	31	139,550	130,567	6.05
	32	139,966	129,687	6.35

Table 2 Comparison of the approaches with 10 **minutes of computation time**

Criteria	Case 1		Case 2		Case 3		Case 4	
	DBS	SLEE	DBS	SLEE	DBS	SLEE	DBS	SLEE
$\overline{Gap}$(%)	3.4	3.1	1	1	1.5	1.6	1.6	1.5
Gap_{max} (%)	5	4.4	1.2	1.2	1.7	1.8	2	2
Gap_{min} (%)	1.8	1.4	0.8	0.9	1.2	1.4	1.2	1.2
$\overline{Gap_{BKS}}$ (%)	0.5	0	0.5	0	0	0.08	0.01	0
$\overline{Waiting}$ ($)	2,430	2,035	4,565	4,480	3,892	4,101	3,792	3,786
$\overline{Cost}$ ($)	65,692	65,341	157,908	157,065	140,909	141,016	132,049	132,034
$\overline{Time}$ (s)	10,854	6,738	4,042	4,111	13,419	12,064	23,231	12,095

Table 3 Results with one regional base

Criteria	Case 1		Case 2		Case 3		Case 4	
	DBS	SLEE	DBS	SLEE	DBS	SLEE	DBS	SLEE
$\overline{Gap}$(%)	5.3	5.1	1.9	1.7	4.5	4.2	5.5	4.6
Gap_{max} (%)	7.4	5.9	2.3	2	5.1	4.7	6.4	5.5
Gap_{min} (%)	3.4	4.2	1.6	1.5	3.7	3.7	4.7	3.9
$\overline{Gap_{BKS}}$ (%)	0.26	0	0.1	0	0.17	0	0.72	0
$\overline{Waiting}$ ($)	3,461	3,188	5,185	4,978	6,748	6,395	7,731	6,785
$\overline{Cost}$ ($)	66,548	66,372	158,275	158,098	143,552	143,296	135,394	134,418
$\overline{Time}$ (s)	22,539	16,078	14,205	7,866	19,072	17,616	17,041	17,633

Table 4 Results with three regional bases

strategy generally outperforms the default strategy in terms of solution quality except for case 3 **with one regional base. It must be noted that the difference between the two strategies is not too significant (less than 1% for all scenarios). This is explained by the fact that the waiting time cost represents less than 6% and therefore, the SLEE branching strategy contribution has an impact only on a small fraction of the overall objective function.** Even if the reduction in terms of average total cost is less than 1%, minimizing average waiting time cost using the SLEE branching strategy is relevant, since this part of solution is directly perceived by drivers and operators of log-loaders.

Gap size increases when the number of bases increases. This was expected since the problem size (constraints and variables) also increases. The results show that the average gap size for large problems is smaller than the one associated with smaller ones (case study 1) (see Tables 3, 4). Bramel and

Simchi-Levi [2] have discussed the impact of problem size on the gap for the set covering formulations of the vehicle routing problem with time windows. They showed that the relative gap between fractional and integer solutions of the problem becomes progressively smaller as the number of customers increases.

The analysis of the relative gap is a bit complex, since it is associated with the lower bound (linear relaxation in our case) and the integer solution. The difference observed between the relative gap of the two strategies is in most cases in favor of the SLEE strategy. Comparing the computational time of both strategies, we notice that the SLEE strategy leads quickly to a good quality solution in most of cases. This reduction in average time to reach best solution in some cases can reach up to 48%.

Finally, as mentioned before in section 4 we use the smaller step-size 10-minute only when the relative gap of 20-minute step-size is less than 1%. Our goal is to use 10-minute step-size to refine the best solution found by 20-minute step-size. However, in practice, cases 2, 3 and 4 were treated by both step sizes, but only two daily SLTSPs associated with one regional base have been improved by the 10-minute step-size. We report that the improvement associated with 10-minute step-size was about 2%.

6 Conclusion

In this paper, we have described a two phase method to solve the weekly Log-Truck Scheduling Problem, that includes a just-in-time delivery policy, a subject of great interest to the forest industry.

The first phase determines the optimal number of full truckloads to deliver from forest areas to wood mills by minimizing the loaded transportation and forest areas costs. This phase yields seven daily SLTSPs. To deal with these, we developed a flow-based model and a specialized branching strategy. Computational experiments show that this specialized strategy provides solutions more rapidly and of higher quality than those produced by the Cplex 12.4.0 default strategy.

Future proposed research directions involve solving the problem in one step, using a Lagrangian relaxation approach by relaxing all constraints that link two successive days. We also plan to produce robust solutions by including stochasticity in trip times, as well as loading and unloading times. We would also like to deal with additional constraints related to driver breaks and shift changes, as well as with the case where the number of home bases equals the number of trucks. We believe that column generation approaches may be beneficial in this context.

References

1. Andersson, G., Flisberg, P., Lidén, B., and Rönnqvist, M. RuttOpt- a decision support system for routing of logging trucks. *Canadian Journal of Forest Research*, 38: 1784-1796,

2008.
2. Bramel, J., and Simchi-Levi, D. On the effectiveness of set covering routing problem with time windows. *Operational Research*, 45(2), 295–301, 1997.
3. Bredström, D., and Rönnqvist, M. Combined vehicle routing and scheduling with temporal precedence and synchronization constraints. *European Journal of Operational Research*, 191(1), 19–31, 2008.
4. D'Amours, S., Rönnqvist, M., and Weintraub, A. Using Operational Research for Supply Chain PLanning in the Forest Products Industry. *Information Systems and Operational Research*, 46(4), 265–281, 2008.
5. El Hachemi, N., Gendreau, M., and Rousseau, L-M. A Hybrid Constraint Programming Approach to the Log-Truck Scheduling Problem. *Annals of Operation Research*, 184(1), 163–178, 2011.
6. El Hachemi, N., Gendreau, M., and Rousseau, L-M. A Heuristic to Solve the Synchronized Log-Truck Scheduling Problem. *Computers and Operations Research*, published on-line, 2011.
7. Eveborn, P., Flisberg, P., and Rönnqvist, M. Laps care- and operational system for staff planning of home care. *European Journal of Operational Research*, 171(3), 962–976, 2006.
8. Eveborn, P., Rönnqvist, M., Einarsdóttir, H., Eklund, M., Lidén, K., and Almroth, M. Operations Research Improves Quality and Efficiency in Home Care. *Interfaces*, 39(1), 18–34, 2009.
9. Flisberg, P., Lidén, B., and Rönnqvist, M. A hybrid method based on linear programming and tabu search for routing of logging trucks. *Computers and Operations Research*, 36(4), 1122–1144, 2008.
10. Gronalt, M., and Hirsch, P. Log-Truck Scheduling with Tabu Search Strategy. *Metaheuristics*, Vol 39, 65–88, 2007.
11. `http://cfs.nrcan.gc.ca/pages/269?lang=en_CA`
12. `http://www.fao.org/forestry/country/57478/en/fin/`
13. Linnainmaa, S., Savalo, J., and Jokinen, O. EPO: A knowledge based system for wood procurement management. *Paper presented at the 7th Annual Conference on Artifical Intelligence*, Montreal, 1995.
14. Murphy, G. Reducing trucks on the road through optimal route scheduling and shared log transport services. *Southern Journal of Applied Forestry*, 27(3), 198–205, 2003.
15. Palmgren, M., Rönnqvist, M., and Värbrand, P. A solution approach for log truck scheduling based on composite pricing and branch and bound. *International Transactions in Operational Research*, 10, 433-447, 2003.
16. Palmgren, M., Rönnqvist, M., and Värbrand, P. A near-exact method for solving the log-truck scheduling problem. *International Transactions in Operational Research*, 11, 447-464, 2004.
17. Rey, P.A., Muñoz, J.A., and Weintraub, A. A Column Generation Model for Truck Routing in the Chilean Forest Industry. *Information Systems and Operational Research*, 47(3), 215-221, 2010.
18. Rönnqvist, M. Optimization in forestry. *Mathematics Programming*, 97, 267–284, 2003.
19. Rönnqvist, M., Sahlin, H., and Carlsson, D. Operative planning and dispatching of forestry transportation. *Report LiTH-MAT-R-1998-18, Linköping University*, Linköping, Sweden.
20. Rönnqvist, M. and Ryan, D. Solving truck dispatch problem in real time. *In Proceedings of the 31st Annual Conference of the Operational Research Society of New Zealand, 31 August - 1 September 1995*, Wellington, New Zealand. The Operational Research Society of New Zealand, Auckland, N-Z. pp. 165–172.
21. Toth, P., and Vigo, D. The vehicle routing problem. *Society for industrial and applied Mathematics*, 2001.
22. Weintraub, A., Epstein, R., Morales, R., Seron, J., and Traverso, P. A truck scheduling system improves efficiency in the forest industries. *Interfaces* 26(4), 1–12, 1996.