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A HYBRID CONSTRAINT PROGRAMMING APPROACH TO A WOOD PROCUREMENT PROBLEM WITH BUCKING DECISIONS

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Abstract In this article, we present a wood procurement problem that arises in Eastern Canada. We solve a multi-period wood supply planning problem, while taking into account bucking decisions. Furthermore, we present a new form of flexibility which allows the harvesting capacity to change from one time period to another. We study the impact of such flexibility upon the harvesting cost. We assess the performance of the problem by comparing it with a variant where the harvesting capacity is fixed during sites' harvesting. To address this problem, we develop a hybrid approach based on both constraint and mathematical programming. In the first phase, we propose a constraint programming model dealing with forest sites harvesting and bucking problems. The result of this model is used as part of an initial solution for the whole problem formulated as a mixed integer model. We test the two versions of the problem on a set of different demand instances and we compare their results.

Keywords Forest bucking problem · Wood-procurement planning · Constraint programming · Hybrid method.

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1 Introduction

Forest industry represents an important economic sector in Canada. It is among the top five contributors to the nation's net trade (Natural Resources Canada, 2014). Forest planning problems cover a wide range of activities: planting, harvesting, road building, transportation, etc. They are the focus of important development to help the sector adapt to new challenges such as environmental issues and tough competition.

Wood procurement planning problems (WPPP) encompass a wide range of activities that provide quantities of wood to processing mills. In this paper, we present a multi-period wood procurement planning problem including bucking decisions for a planning horizon of one year. Given a list of forest sites to harvest, we must decide which sites to harvest in each period and the products that should be obtained from each period. We also consider the allocation of harvested products to different wood-processing facilities. Our main goal is to find a near-optimal wood-procurement plan for large instances. Through our collaboration with FPInnovations, the method will be used to support Eastern Canadian forest companies.

The proposed method is based on two phases. In the first phase, we propose a constraint programming (CP) model. It aims to determine a harvesting schedule for each forest site and the allocations of bucking priority lists to different tree species, in order to minimize the harvesting cost. In the second phase, we use the solution of the CP model as part of the initial solution of the whole mixed integer problem (MIP). This phase deals with the harvesting activities, the storage decisions as well as the wood flow between forest areas and wood mills. Then, we compare the current problem with the variant presented in Dems et al. (2014), where the harvesting capacity is unchanged during harvesting from a time period to another.

This paper is organized as follows. Section 3 describes the problem. Section 2 presents an overview of the literature. Next in Section 4, we present our solution approach. The data used in our tests and the computational results are introduced in Section 5. Finally, Section 6 presents concluding remarks and some research perspectives.

2 Wood Procurement Planning Problem

To date, solving the wood procurement planning problem (WPPP) relies heavily on computer-aided modeling of operations. The majority of these models have concentrated on managing individual elements of the wood supply chain (WSC) such as harvesting and crew scheduling, machine location, transportation, and storage management. Even, there is an apparent need to considering more integrated wood procurement planning models due to the recent changes in the forest industry, considering the requirements of different elements of the wood supply into the same model is still challenging.

Bucking problems are well studied in the literature (Kivinen (2007), Kivinen (2006), Laroze (1993)). However, few models of WPPP including bucking decisions have been reported in the field of wood procurement planning. Dems et al. (2013) proposed an annual procurement plan that incorporates bucking and transportation activities. The problem addressed in this paper presented a mono-periodic plan that did not deal with variability of the demand and supply according to time periods and did not consider the harvest scheduling decisions, these issues are taken into account in the problem we address in this paper. The bucking patterns are generated using a priority-list approach. The model includes a harvesting cost function that considers the nonlinearity of the harvester productivity function;which is important in forest management (Arce et al., 2002).

Several models dealing with WPP have been developed using various types of operations research techniques (Weintraub et al. (2007), Rönnqvist (2003), Björndala et al. (2012)). CP has recently emerged as a research area that combines modeling and solving various combinatorial problems, especially in areas of planning and scheduling (Baptiste et al., 2001). Hybridization approaches combining CP and other programming techniques have been successfully applied to different problems (Hooker (2005), Sakkout et al. (2000)). The reader is referred to the work of Milano (2004), for more details about decomposition methods involving CP.

In the forestry context, a CP based hybrid approach was developed to solve the log-truck scheduling problem (Flisberg et al., 2007). The authors reported that CP can easily deal with the synchronization of trucks and log-loaders. El Hachemi et al. (2011) proposed recently a solution method based on constraint programming and mathematical programming for another variant of log-truck scheduling problem. The decisions deal with scheduling the transportation of logs between forest areas and wood mills, as well as routing the fleet of vehicles. The problem aims at minimizing the total cost of the non-productive activities such as the waiting time of trucks and forest log-loaders in addition to the empty driven distance of vehicles. To our knowledge, no studies addressing the bucking driven wood procurement problem and using constraint based approaches exist.

This paper presents a hybrid approach to solve a wood procurement problem with bucking decisions. The method combines constraint and mathematical programming (CP/MIP approach). First, we propose a constraint programming model to determine a harvesting schedule for every forest site as well as to deal with the assignments of bucking priority lists to different species. Then, the CP solution is communicated to a MIP model, which deals with the whole problem including procurement activities.

3 Problem Statement

In order to understand the specificities of the wood procurement problem addressed in this paper, we discuss in this section the Eastern forestry practices during activities from bucking tree stems to delivering products to mills. Fig. 1 illustrates the general framework of the problem, it resumes the main activities from the wood supply chain included in this problem, then we describe step by step these activities in the following subsections.

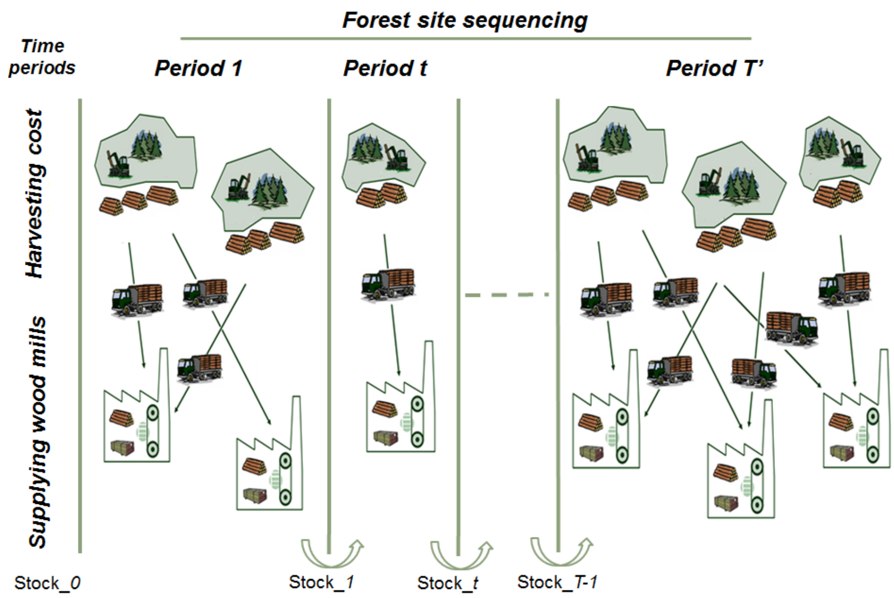

Fig. 1 Framework of the integrated multi-period wood procurement problem (Dems et al., 2014)

3.1 Harvesting activities

The forest is divided into a large set of heterogeneous forest sites. We assume that these sites are predefined in a higher-level of the harvesting planning problem and are accessible via the road network. Their management is centralized and done by the same multifacility forest company. Predefined sets of adjacent sites constitute sectors.

Bucking. Bucking is the operation of cutting tree stems into smaller pieces (logs) to be used in further industrial processing (Arce et al. (2002), Kivinen (2007)). When this operation is done directly in forest using cut-to-length machinery such as harvesters and forwarders, we talk about cut-to-length bucking. The harvester cross-cuts different logs following the cutting instructions. In this context, bucking instructions (patterns) are not determined by the onboard computers. Therefore, generated bucking patterns must be simple and easy for operators to implement.

The priority-list bucking approach described in Dems et al. (2013) is used to generate simple patterns. In this approach, a bucking priority list is a sequence of allowable logs obtained from a stem, generated according to practical rules defined by the forest company. In fact, the position of a product in the priority list is defined by its commercial value, length, and minimum small end diameter (MSED). The lowest priority is assigned to the log, generally a pulp log, with the smallest MSED and the shortest length. Then, the bucking algorithm allocates logs to each stem section using an appropriate priority list. A bucking priority list is assigned to each species. To predict the yield products, we use the software FPInterface. This simulation tool, designed by FPInnovations, is used to simulate different activities in the forest supply chain. The simulation module can predict the yield products from the application of a given bucking priority list to a sample of trees.

Harvesting cost. The specifications of forest sites (blocks) in Eastern Canada leads to a number of important log types that can be harvested in the same site. A reduction of 1%–4% in the harvester productivity (respectively 3%–7% in the forwarder productivity) is generated by harvesting a new log type in a block (see Gingras et al. (2002), Brunberg et al. (2001)). The important number of different logs types bucked per block increases the harvesting cost and leads to complex instructions for the log makers. In this work, we also consider the effect of productivity decrease in harvesting machinery on the harvesting cost. This latter increases nonlinearly with the number of different products bucked per block and decreases with their average length. For more details about the nonlinear formulation of the harvesting cost and the priority list approach, the reader is referred to Dems et al. (2013).

Forest sites sequencing. The proposed model addresses the scheduling of forest sites' harvesting over time periods. These decisions are usually included in tactical planning. In this paper, we consider a set of schedules for each forest site. To generate them:

- We consider different production categories corresponding to five different harvesting capacities (m^3/period), which are related to the production capacities of a number of harvesting teams and their corresponding equipments defined by FPInnovations.
- For a given forest site harvested according to the largest (resp. the smallest) harvesting capacity, we take the ceiling of the division of its standing timber by the associated harvesting capacity. This gives us the maximum (resp. the minimum) duration, which is the number of periods needed to harvest the whole site.
- We consider all the possible duration between the maximum and the minimum values, this represents the harvesting categories.
- We associate to each category, a set of possible harvesting sequences when beginning the harvesting in different time periods of the planning horizon. This represents the set of schedules.

3.2 Supply activities

We consider a set of geographically distributed mills to supply, each has an annual demand expressed in terms of different volumes of products. A product is defined by a given product type (length, MSED) and species. After being bucked, logs are hauled to roadside and stored in different piles until they are transported to mills. Even though, there is no limitation on the volume kept in the forest; it is not desirable that too many products be stored at the roadside since they loose freshness. A unit storage cost in the forest, representing the quality deterioration of logs in the forest, is considered. The transportation cost depends on the distance between forest sites and mills as well as on products. There is a transportation capacity limit on the total volume transported in each period.

A part of each delivery is used to meet mills demand, and the remainder is kept in the storage areas, with an associated stock cost which depends on time periods. Each mill has a storage capacity and a stock cost corresponding to the quality deterioration specially in summer. This storage cost in mills is slightly higher than the one in the forest. We do not allow the exchange of timber between different mills.

4 The CP/MIP approach

In this problem, two blocs of decisions are considered. Each of these decisions relies on a different side of the procurement problem. The first are dealing with the harvesting activities. The output of this bloc of decisions represents the supply or the source. The second represents the procurement activities i.e. allocation, transportation and inventory management.

Our first attempt was to solve the whole problem by pure CP approach. Pure CP take surprisingly long time to run. We tried to analyze which part of the problem takes longer time to run: the harvesting problem (containing the binary variables dealing with harvest scheduling and bucking decisions) or the procurement problem (containing the continuous variables dealing with the transportation and inventory management). When running separately these problems using pure CP approach, we noticed that the procurement problem takes a surprisingly long time to run. This observation as well as the hierarchical nature of this decisions making process motivates the CP/MIP approach.

This approach combines constraint and mathematical programming. The constraint programming model deals with the assignments of bucking priority lists to different species and the harvest sequencing of the forest sites. These harvesting decisions are determined only once. Then, they are communicated to the whole model including the harvesting and the procurement activities, expressed as a mixed integer linear model. The fixed variables from the CP model are communicated to the whole MIP problem and used to find an initial solution for it. This solution will be improved during the search process of the solver.

The details of the CP formulation are given in Section 4.1 and the MIP model is presented in Section 4.2. The following parameters are used in the two models:

Parameters

B	Set of forest sites (blocks);
P	Set of product types;
E	Set of species;
E_b	Set of species in site b;
R	Set of priority lists;
I	Set of schedules;
I_b	Set of schedules for site b;
T	Set of time periods, $T = \{0, 1, ..., \|T\| - 1\}$;
V_t^H	Harvesting capacity in time period t (m^3);
V_b	Volume of timber available in forest site b (m^3);
V_{br}^{ep}	Volume of product type p, $\forall p \in P$, available when bucking species e of forest site b, according to priority list r;
V_{Pc}^{max}	Largest volume of the production categories;
P^{Tr}	Penalty cost corresponding to the change of harvesting capacity between two adjacent time periods;
P^{Pl}	Penalty cost corresponding to unused production capacity (production loss);
Nb^{max}	Maximum number of forest sites to harvest in each period;
ϕ_b	Minimum fraction to harvest from a forest site;
a_i^t	Coefficient used to extract information from schedule i. Takes value 1 if harvesting occurs in period t, 0 otherwise.

4.1 The CP model

This section presents the detailed formulation of the constraint programming model (CP model) which is implemented to find a solution to the harvesting problem without procurement activities. The following parameters and variables are used:

CP_Parameters

C_{pe}^D	Unit cost corresponding to unsatisfied demand of product type p, $\forall p \in P$, species e;
C_{ber}^H	Unit harvesting cost for each priority list r applied to each species e in a given forest site b;
D_{pe}^t	Total mills' demand for product type p, $\forall p \in P$, of species e in period t (m^3);

M Large number, for example equal to the total harvesting capacity.

CP_Variables

- $\hat{r}^b_e$ Returns the priority list r assigned to the species e of forest site b, this variable takes an integer value defined on finite domain $[0, |R| - 1]$;
- $\hat{z}_b$ Corresponds to the schedule assigned to the site b, this variable takes an integer value defined on a finite domain $[0, |I_b| - 1]$;
- $\hat{q}^t_b$ Defines the harvesting capacity used in time period t to harvest site b, this variable takes an integer value defined on finite domain $[0, V^{max}_{Pc}]$;
- $\hat{y}^t_b$ Corresponds to the fraction of site b, harvested in time period t, continuous variable defined on domain $[0, 1]$;
- $\hat{x}^t_{pe}$ Represents the unsatisfied demand of product type p, $\forall p \in P$, species e, in period t, continuous variable defined on domain $[0, M]$;

CP_Model

$$\textbf{Min} \quad C^H + \sum_{t\in T} \sum_{e\in E_b} \sum_{p\in P} C^D_{pe} \hat{x}^t_{pe} + Penal_1 + Penal_2$$

subject to

$$\sum_{b\in B} \hat{y}^t_b V_b \leq V^H_t \qquad \forall t \in T \tag{1}$$

$$\hat{y}^t_b V_b - \hat{q}^t_b \leq 0 \qquad \forall b \in B \text{ and } \forall t \in T \tag{2}$$

$$\sum_{b\in B} a^t_{[\hat{z}_b]} \leq Nb^{max} \qquad \forall t \in T \tag{3}$$

$$\sum_{t\in T} \hat{y}^t_b = 1 \qquad \forall b \in B \tag{4}$$

$$\hat{y}^t_b - a^t_{\hat{z}_b} \leq 0 \qquad \forall b \in B \text{ and } \forall t \in T \tag{5}$$

$$\hat{y}^t_b - \phi_b a^t_{\hat{z}_b} \geq 0 \qquad \forall b \in B \text{ and } \forall t \in T \tag{6}$$

$$\hat{q}^t_b - V^{max}_{Pc} a^t_{\hat{z}_b} \leq 0 \qquad \forall b \in B \text{ and } \forall t \in T \tag{7}$$

$$\sum_{b\in B} \hat{y}^t_b V^{ep}_{b[\hat{r}^b_e]} + \hat{x}^t_{pe} \geq D^t_{pe} \qquad \forall e \in E, \forall p \in P \text{ and } \forall t \in T \tag{8}$$

The objective function minimizes the global operational costs. The first term C^H is the total harvesting cost, given in equation A:

$$C^H = \sum_{b\in B} \sum_{e\in E_b} \sum_{p\in P} (n_b)^\gamma V^{ep}_{b[\hat{r}^b_e]} C^H_{be[\hat{r}^b_e]} \tag{A}$$

where
$n_b = \sum_{e \in E_b} \sum_{p \in P} \left[V^{ep}_{b[\hat{r}^b_e]} \neq 0 \right]$, is the number of different products obtained in a forest site b;
$\gamma < 1$, is an empirical constant determined by FPInnovations.

The second term of the objective function aims to penalize unsatisfied demand. This term generates better solutions by matching the total demand as much as possible .

$Penal_1$ corresponds to a penalty for changing the production capacity between two adjacent periods when harvesting occurs. In practice, a change of production capacity between two adjacent periods when harvesting occurs, means a change (relocations or new assignments) of crews and their equipments between these periods, which generates an additional cost due to transfer or transportation costs. This penalty is calculated in the objective function as follows:

$$Penal_1 = \sum_{b \in B} \sum_{t \in T \setminus \{|T|-1\}} P^{Tr} |\hat{q}^{t+1}_b - \hat{q}^t_b| a^t_{[\hat{z}_b]} a^{t+1}_{[\hat{z}_b]} \tag{B}$$

Finally, $Penal_2$ corresponds to a penalty for the production loss due to harvesting less than the allocated harvesting capacity. This is penalized in the objective function using the cost term given in equation C:

$$Penal_2 = \sum_{b \in B} \sum_{t \in T} P^{Pl} (\hat{q}^t_b - \hat{y}^t_b V_b) \tag{C}$$

Constraints 1 limit the total volume harvested per period. Constraints 2 specify that the harvested volume does not exceed the used harvesting capacity. Constraints 3 limit the number of forest sites in which harvesting can occur during a period. Constraints 4 mean that the total summation of proportions of each harvest area are is 1.

Constraints 5 to 7 ensure the continuity of harvesting activities, which means that either a site is not harvested or it is fully harvested without interruption, even though it is possible to change the production capacity used between time periods. Constraints 8 aims at satisfying the demand of each product in each period, and the slack variables $\hat{x}^t_{pe}$ guarantee feasible solutions.

4.2 The MIP model

We present in this section a mixed-integer linearized mathematical formulation (P) that has been implemented to solve the problem. The parameters and the variables used are as follows:

MIP_Parameters

P_r Set of products in bucking priority list r;

U Set of mills;

V_t^T Transportation capacity in time period t (m^3);
V_u^S Stock capacity of mill u (m^3);

C_{bern}^H Unit harvesting cost for each priority list r applied to each species e in a given forest site b;
C_{bupe}^T Unit transportation cost between forest sites b and mill u for product type p, $\forall p \in P$, of species e ($\$/m^3$);
C_t^{SF} Unit stock cost, in forest, during time period t ($\$/m^3$);
C_t^{SU} Unit stock cost, in mill u, during time period t ($\$/m^3$);
C_{uep}^D Unit cost corresponding to unsatisfied mill's u demand of product type p, $\forall p \in P$, species e;

$maxN$ Maximum number of different products that can be harvested from a block;
n Number of different products that can be harvested from a block, $n \in N = \{1, .., maxN\}$;
D_{uep}^t Demand at mill u for product type p, $\forall p \in P$, of species e in period t (m^3);
φ_r Number of different product types in the priority list r.

MIP_Variables

$$w_b^n = \begin{cases} 1, & \text{if } n \text{ different products are obtained from site } b; \\ 0, & \text{otherwise.} \end{cases}$$

$$\tilde{w}_{ber}^n = \begin{cases} 1, & \text{if bucking priority list } r \text{ is applied to species } e \text{ from site } b, \\ & \text{when } n \text{ different products are obtained from } b; \\ 0, & \text{otherwise.} \end{cases}$$

$$z_b^i = \begin{cases} 1, & \text{if site } b \text{ is allocated to schedule } i; \\ 0, & \text{otherwise.} \end{cases}$$

$$\tilde{q}_b^t = \begin{cases} |q_b^{t+1} - q_b^t|, & \text{if harvesting site } b, \text{ occurs in periods } t+1 \text{ and } t; \\ 0, & \text{otherwise.} \end{cases}$$

q_b^t Production capacity used to harvest site b in time period t;
Absolute value of the difference in production capacities used in site b, between periods $t+1$ and t;

y_b^t Fraction of site's b timber, harvested in time period t;
$\tilde{y}_{ber}^t$ Fraction of volume of species e from b, when bucked, in period t using priority list r;

x^{bt}_{uep} Flow of product type p, $\forall p \in P$, species e from site b to mill u in period t (m^3);
$\tilde{x}^{t}_{uep}$ Unsatisfied demand of product type p, $\forall p \in P$, species e, at mill u, during time period t;

s^{t}_{bep} Stored volume of product type p, $\forall p \in P$, species e, in site b, at the end of period t (m^3);
$\tilde{s}^{t}_{uep}$ Stored volume of product type p, $\forall p \in P$, species e, in mill u, at the end of period t (m^3).

MIP_Model

$$(P) \quad \text{Min} \sum_{b\in B}\sum_{e\in E_b}\sum_{r\in R}\sum_{p\in P_r}\sum_{n\in N} C^{H}_{bern} V^{ep}_{br} \tilde{w}^{n}_{ber} + \sum_{t\in T}\sum_{b\in B}\sum_{e\in E_b}\sum_{p\in P}\sum_{u\in U} C^{T}_{bupe} x^{bt}_{uep}$$
$$+ \sum_{t\in T}\sum_{b\in B}\sum_{e\in E_b}\sum_{p\in P} C^{SF}_{t} s^{t}_{bep} + \sum_{t\in T}\sum_{e\in E_b}\sum_{p\in P}\sum_{u\in U} C^{SU}_{t} \tilde{s}^{t}_{uep}$$
$$+ \sum_{t\in T}\sum_{b\in B}\sum_{e\in E_b}\sum_{p\in P}\sum_{u\in U} C^{D}_{uep}\tilde{x}^{t}_{uep} + \sum_{t\in T}\sum_{b\in B} P^{Tr}\tilde{q}^{t}_{b} + \sum_{t\in T}\sum_{b\in B} P^{Pl}(q^{t}_{b} - y^{t}_{b}V_{b})$$

subject to

Bucking activities

$$\sum_{n\in N} w^{n}_{b} = 1 \qquad \forall b \in B \tag{9}$$
$$\sum_{e\in E_b}\sum_{r\in R} \varphi_r \tilde{w}^{n}_{ber} - n w^{n}_{b} = 0 \qquad \forall b \in B \text{ and } \forall n \in N \tag{10}$$
$$\sum_{n\in N}\sum_{r\in R} \tilde{w}^{n}_{ber} = 1 \qquad \forall b \in B \text{ and } \forall e \in E_b \tag{11}$$
$$\sum_{n\in N} \tilde{w}^{n}_{ber} - \sum_{t\in T} \tilde{y}^{t}_{ber} = 0 \qquad \forall b \in B, \forall e \in E_b, \text{ and } \forall r \in R \tag{12}$$

Forest sites scheduling activities

$$\sum_{b\in B} y^{t}_{b} V^{b} \le V^{H}_{t} \qquad \forall t \in T \tag{13}$$
$$\sum_{i\in I_b} z^{i}_{b} = 1 \qquad \forall b \in B \tag{14}$$
$$y^{t}_{b} V^{b} - q^{t}_{b} \le 0 \qquad \forall b \in B \text{ and } \forall t \in T \tag{15}$$
$$\sum_{r\in R} \tilde{y}^{t}_{ber} - y^{t}_{b} = 0 \qquad \forall b \in B, \forall e \in E_b \text{ and } \forall t \in T \tag{16}$$
$$\sum_{b\in B}\sum_{i\in I_b} a^{t}_{i} z^{i}_{b} \le Nb^{max} \qquad \forall t \in T \tag{17}$$

$$\sum_{i \in I_b} a_i^t z_b^i - y_b^t \leq 1 - \phi_b \qquad \forall b \in B \text{ and } \forall t \in T \tag{18}$$

$$y_b^t - \sum_{i \in I_b} a_i^t z_b^i \leq 0 \qquad \forall b \in B \text{ and } \forall t \in T \tag{19}$$

$$q_b^t - V_{Pc}^{max} \sum_{i \in I_b} a_i^t z_b^i \leq 0 \qquad \forall b \in B \text{ and } \forall t \in T \tag{20}$$

$$\tilde{q}_b^t - V_{Pc}^{max} \sum_{i \in I_b} a_i^t a_i^{t+1} z_b^i \leq 0 \qquad \forall b \in B \text{ and } \forall t \in T \setminus \{|T|-1\} \tag{21}$$

$$\tilde{q}_b^t - \bar{q}_b^t \leq 0 \qquad \forall b \in B \text{ and } \forall t \in T \tag{22}$$

$$\bar{q}_b^t - V_{Pc}^{max}(1 - \sum_{i \in I_b} a_i^t a_i^{t+1} z_b^i) - \tilde{q}_b^t \leq 0 \qquad \forall b \in B \text{ and } \forall t \in T \setminus \{|T|-1\} \tag{23}$$

$$\bar{q}_b^t \geq q_b^{t+1} - q_b^t \qquad \forall b \in B \text{ and } \forall t \in T \setminus \{|T|-1\} \tag{24}$$

$$\bar{q}_b^t \geq q_b^t - q_b^{t+1} \qquad \forall b \in B \text{ and } \forall t \in T \setminus \{|T|-1\} \tag{25}$$

Procurement activities

$$s_{bep}^t = \sum_{r \in R} \tilde{y}_{ber}^t V_{br}^{ep} - \sum_{u \in U} x_{uep}^{bt} + s_{bep}^{t-1} \quad \forall b \in B, \forall e \in E_b, \forall p \in P \text{ and } \forall t \in T \backslash \{0\} \tag{26}$$

$$\tilde{s}_{uep}^t = \tilde{s}_{uep}^{t-1} + \sum_{b \in B} x_{uep}^{bt} - D_{uep}^t + \tilde{x}_{uep}^t \quad \forall u \in U, \forall e \in E_b, \forall p \in P \text{ and } \forall t \in T \backslash \{0\} \tag{27}$$

$$\sum_{e \in E} \sum_{p \in P} \tilde{s}_{uep}^t \leq V_u^S \quad \forall u \in U \text{ and } \forall t \in T \tag{28}$$

$$\sum_{b \in B} \sum_{e \in E_b} \sum_{p \in P} \sum_{u \in U} x_{uep}^{bt} \leq V_t^T \quad \forall t \in T \tag{29}$$

$$w_b^n, \tilde{w}_{ber}^n, z_b^i \in \{0,1\} \quad \forall b \in B, \forall r \in R, \forall e \in E, \forall p \in P \text{ and } \forall n \in N \tag{30}$$

$$\bar{q}_b^t, \tilde{q}_b^t, q_b^t \in \{0, 1, 2 ... |V_{Pc}^{max}|\} \quad \forall b \in B \text{ and } \forall t \in T \tag{31}$$

$$x_{uep}^{bt}, \tilde{x}_{uep}^t, s_{bep}^t, \tilde{s}_{uep}^t, y_b^t, \tilde{y}_{ber}^t \geq 0 \quad \forall b \in B, \forall e \in E, \forall p \in P, \forall u \in U \text{ and } \forall t \in T \tag{32}$$

The objective function minimizes the harvesting cost, the transportation cost, the storage cost in forest and at mills respectively. The unit harvesting cost C_{bern}^H is calculated using the approximation proposed in Dems et al. (2013). The fifth term in the objective function corresponds to a sufficiently large cost for not satisfying the demand. Then, the sixth term penalizes the change of production capacities between two adjacent periods when harvesting occurs. Finally, the last term corresponds to a penalty for unused harvesting capacity.

Constraints 9 and 10 count the number (n) of different products (logs) harvested in each forest sites. Constraints 11 ensure that only one n and one bucking list per species are used in each forest site. Constraints 12 specify that the sum of the volume proportions of a species using a bucking list r is equal to one if this priority list is assigned to it and zero otherwise.

Constraints 13 through 25 deal with forest sites scheduling. Constraints 13 limit the total volume harvested per period. Constraints 14 correspond to the restriction that only one schedule is allocated to each forest site. Constraints 15 specify that the harvested volume must not exceed the harvesting capacity used. Constraints 16 mean that the proportion of the harvested volume from each species is equal to the proportion harvested from the forest site volume. This is an approximation of the real problem, since we consider that the species are uniformly distributed in the forest sites. Constraints 17 limit the number of forest sites in which harvesting can occur during a period.

Constraints 18 to 20 ensure the continuity of harvesting activities of a forest site once began, which means that if a site is harvested, it is fully harvested. No interruption in harvesting is allowed, even if it is possible to change the production capacity used.

Constraints 21 through 25 are used to calculate the change of production capacities between two adjacent periods when harvesting occurs. The harvesting capacity can be increased or decreased that is why we use the variables $\bar{q}_b^t$ to get the absolute value of its change between two adjacent periods. This change is penalized in the objective function through P^{Tr}.

Constraints 26 to 29 define the procurement activities. Constraints 26 and 27 represent the flow conservation constraints at the forest and mills. In constraints 27, the slack variables $\tilde{x}_{uep}^t$ are considered to guarantee feasible solutions, and the cost C_{uep}^D for slack variables, assures that the demand will be satisfied as much as possible. Constraints 28 correspond to the maximum storage volume per period in each mill. Constraints 29 limit the total transportation capacity. Constraints 30 state that the variables z_b^i, w_b^n and $\tilde{w}_{ber}^n$ are binary. Constraints 31 ensures that variables $\bar{q}_b^t$, $\tilde{q}_b^t$ and q_b^t are integers. Constraints 32 are non-negativity constraints.

5 Experimental Results

The case study was provided by the Forest Engineering Research Institute of Canada (FPInnovations). Thirty mixed forest sites in Eastern Canada are eligible for harvesting during the year. Furthermore, the case study involves five wood mills. Five log-types, varying in terms of length and MSED are considered for each of the five tree species.

For the computational experiments, hypothetical but realistic demand instances are generated on the basis of one instance of the firm's real data from FPInnovations, and the typical situation of several forest companies in Eastern Canada. In each instance, we vary the volume of the product mix required per mill. The mills' demand are expressed in terms of volume of products, per periods of two weeks. We make sure that the mills' production capacities are respected in each period. Moreover, we consider an upper bound on every product, which represents the yield of this product when it is considered as the first product in the bucking priority lists used. The average demand is 5% to 8% less than the total quantity of standing timber, which represents low-value small-diameter logs and branches.

Experimentations are done on a 2-dual core Intel Xeon processeurs 2.1 GHz, 5 GB of memory. The MIP model is solved using the commercial LP package Cplex v12.5 via its Concert Technology C++ platform. The CP model is solved using CP-optimizer v12.5.

The CP model encompasses 2220 variables and 4262 constraints. The MIP model contains 243832 variables with 65811 binary and 35312 constraints. We note that CP can easily model some practical aspects that were very difficult to model and solve using mathematical programming. Through the use of *element constraints*, the number of variables and constraints decreased in the CP model. In fact, when we consider the MIP formulation, we notice that the binary variables w_b^n and $\tilde{w}_{ber}^n$, for example, are no more necessary when using the element variables $V_{b[\hat{r}_e^b]}$, $C_{be[\hat{r}_e^b]}^H$ and $a_{[\hat{z}_b]}^{t+1}$.

To assess the performance of the hybrid approach, we compare it with the basic approach where the model is solved directly using Cplex. Each test is run using the basic and the hybrid approach. In Table 1, we represent the demand instances used to test the performance of the model (Ins) for both approaches. For the basic approach (Cplex), we report the value of the problem's solution (Val_46) and its corresponding optimality gap (Gap_46) after 46 h of computation time (execution time due to memory capacity limit). For our CP/MIP approach, we present the value of the problem's solution obtained at about 5% of optimality gap (Val_5) and the time consumed to obtain it ($Time\_5$).

After several test with different parameters of Cp_optimizer, we kept the ones that gave the best results. The search technique chosen uses a depth-first search, which is restarted after a certain number of failures. The parameter *RestartGrowthFactor* controls the increase of this number between restarts. In our case, we set this parameter to 1. The initial fail limit is controlled with the parameter *RestartFailLimit*, that we set to 14 in our settings. For tuning the search, we used three search phases. These phases force the search to fix the decision variables $\hat{z}_b$ first, second the variables $\hat{y}_b^t$, then the variables $\hat{r}_e^b$, before instantiating any other variable in the model. In addition, we set a limit time of 30 minutes since the CP solution did not change a lot after this limit. We kept the fail limit to its default value.

For Cplex settings, we used two phases to control the search process. For the first one (the main part), the default setting for branching are used, except one parameter which is "polishing": after an initial solution is available, the heuristic branching parameter (polishing) is used to yield better solutions in situations where good solutions are otherwise hard to find. Since Polishing is time intensive, we use it until a solution with a gap limit of less than 6% is found, then we set up a second short phase of conventional branch & cut using a parameter that emphasizes moving best bound. This parameter allows the user controlling the trade-offs between speed, feasibility, optimality, and moving bounds in a MIP. At the end of the first search phase (using polishing), we found a solution with a gap close to 5%, so we tried to place greater emphasis on proving optimality through moving the best bound value by using the Best Bound parameter.

Table 1 Comparison of the solution approaches

	Cplex		CP/MIP approach	
Ins	Val_46 (10^7\$)	Gap_46 (%)	Val_5^* (10^7\$)	$Time\_5$ (h)
0	-	-	2.226	7.66
1	2.199	9.08	2.110	10.46
2	2.277	8.14	2.208	14
3	-	-	2.082	13.78
4	-	-	2.152	7.85
5	2.234	8.29	2.164	9.60
6	-	-	2.262	11.92
7	-	-	2.208	7
8	-	-	2.134	7.76
9	-	-	2.209	14
10	2.240	8.50	2.179	14
11	2.338	11.20	2.192	14

-: no solution after 46 h *: solution at about 5% of gap

The experiments show that our approach successfully finds good solutions for all the problem's instances within 11 h of average CPU Time. However, it was impossible to find any solution in that time limit using Cplex directly (see Table 1). The CP/MIP approach outperforms Cplex in terms of solutions quality with an average gap of 5% compared to 9%, and computation time of 11 h compared to 46 h for the solved instances by Cplex directly. We note that it is very difficult for Cplex to find a first integer solution. We report also that the CP provided a good enough solution, to allow Cplex to find a first solution of an average gap less than 15% for all instances.

We notice that in almost all the instances the CP solution value did not change after thirty minutes of computation time which explains the chosen time limit. However, the majority of change in the problem solution value and quality obtained by Cplex, is done in the 14 h of computation time. The remainning time is used only to prove optimality.

In order to study the impact of the harvesting capacity change during time periods on the total cost, we compare the problem's solution to the solution of the fixed capacity variant presented by Dems et al. (2014). In the work of Dems et al. (2014), the harvesting capacity is kept unchanged between each site's harvesting duration. We run that model using the new demand instances. Moreover, we generate schedules corresponding to the new planning horizon. We also add the penalty cost P^{Pl} corresponding to unused production capacity to make a fair comparison.

We report, in Fig 2, the total costs in dollars of both the fixed production capacity problem (Fix_Cap) and the one with variable capacity (Var_Cap).

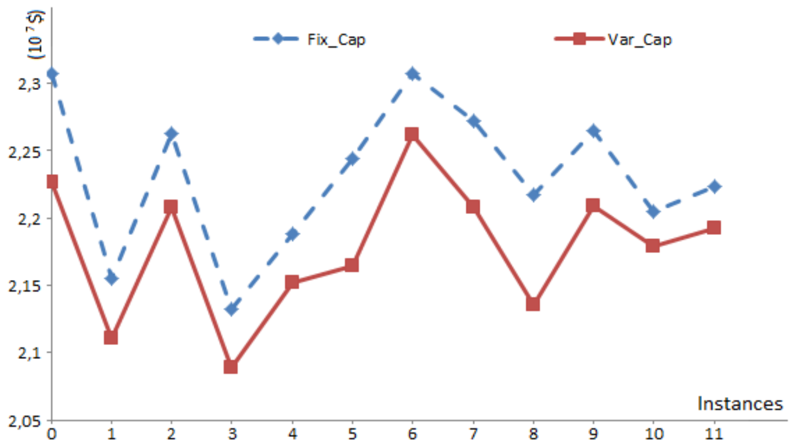

Fig. 2 Total cost of fixed and variable harvesting capacity scenarios

Looking at this figure, we observe, as expected, that using a fixed capacity during harvesting gives a higher total cost than changing it, for all the instances considered.

To deeply analyze the results, we calculate in table 2, the relative percentage change in the total cost (C^{tot}) for the fixed harvesting capacity variant in comparison to the current variant. Moreover, we calculate this change in each term composing the total costs: the harvesting cost (C^{H}), the transportation cost (C^{T}), the production loss cost (C^{Pl}), the total stock cost (C^{S}), the stock cost at mills (C^{SU}), the stock cost at forest (C^{SF}) and the cost related to unsatisfied demand (C^{Slk}).

Table 2 Percentage change of Fix_Cap problem comparing to the Var_Cap one

Ins	C^{tot}	C^{H}	C^{T}	C^{Pl}	C^{S}	C^{SU}	C^{SF}	C^{Slk}
0	3.65	1.33	1.45	48.56	8.34	5.44	11.79	6.91
1	2.11	1.55	0.12	5.43	12.74	2.75	23.71	-2.63
2	2.48	0.90	-3.84	55.91	11.89	-0.34	27.55	3.21
3	2.06	1.82	-0.79	12.08	13.79	-1.28	29.91	-0.62
4	1.67	-0.59	0.70	25.13	-1.18	3.74	-5.93	22.28
5	3.71	1.36	-0.77	60.92	10.00	1.19	19.56	4.71
6	2.00	0.72	0.10	79.06	3.78	3.53	4.04	1.41
7	2.91	0.88	-1.66	78.27	5.98	1.90	9.71	5.67
8	3.82	1.42	-0.21	46.71	9.91	3.38	17.66	9.26
9	2.54	0.68	-0.37	67.09	8.46	0.66	16.95	1.49
10	1.07	1.54	0,82	39.53	10.59	-2.82	25.62	-8.57
11	1.33	-0.66	-0,16	74.61	-0.12	-1.37	1.30	7.11
AVG	**2,45**	**0,91**	**-0,39**	**49,44**	**7,85**	**1,40**	**15,16**	**4,18**

As expected, the production loss cost(C^{Pl}) is the most important factor in the total cost increase. The fixed harvesting production allows a (C^{Pl}) increase of about 49, 4%. We notice that cost increase is not affected by changes in transportation and harvesting costs. We observe no more than (0, 91%) in average of harvesting cost increase. We even report a transportation cost decrease in the Fix_Cap variant of about 0, 39% in average. This can be explained by the fact that the considered harvesting and transportation costs are similar in both variants of the problem. The total harvesting cost depends very much on the available timber volume which is the same in both versions. Respectively, the transportation cost is related to distance and products which are not affected much by the capacity change. Finally, we note that there is a slight increase in the inventory cost kept in the forest. It seems that the Fix_Cap model favors storing excess harvesting volumes rather than forest to loosing production.

To conclude the comparison between the two variants, we remark that using a fixed capacity during harvesting instead of a variable one increases the total cost. Nevertheless, such increase is not important (about 2, 45% in the total cost). To explain this, we believe that the fact that the total harvesting capacity established is very close to the available standing timber, lets the flexibility of changing the capacity very restricted. Another reason is due to the fact that the production loss cost (C^{Pl}) represents between 3% to 4% of the total cost which explains why its impact on cost change is not considerable.

6 Conclusion

We presented in the paper a wood procurement problem that arises in the Eastern Canadian forest context. The model coordinates several activities involved in the wood supply chain such as harvesting, transportation, storage in the forest, and storage at the mill terminals, in order to decrease the total cost.

Since the resulting mixed-integer problem is large, Cplex was not able to solve any test. We proposed a hybrid approach based on both constraint and mathematical programming, in order to solve all the problem instances in realistic time limit.

Computational experiments showed that the proposed approach provides solutions under 5% of optimality gap for all the problem instances, within a reasonable time limit (an average of 11 h). In addition, we presented a form of flexibility, that allows production capacity change during harvesting. We compared this variant with the fixed capacity problem. We demonstrated that this flexibility has slightly decreased the total cost (a decrese of about 2, 45%). We believe that it would result in more benefit if the total harvesting capacity will be very different from the standing timber.

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References

Natural Resources Canada. *http://www.nrcan-rncan.gc.ca*, 2014.

Arce, J.E., Carnieri, C., Sanquetta, C.R. and Filho, A.F. A forest-level bucking optimization system that considers customerś demand and transportation costs. *Forest Science.* **48(3)** 492–503, 2002.

Baptiste, P., C. LePape, and W. Nuijten. Constraint-Based Scheduling. *Kluwer Academics Publishers*, 2001.

Beaudoin, D., Frayret, J.-M., and LeBel, L. Hierarchical forest management with anticipation: An application to tactical-operational planning integration. *Canadian Journal of Forest Research.* **38(8)** 2198–2211, 2008.

Björndala, T., Herrerob, I., Newmanc, A., Romerod, C., and Weintraub, A. Operations research in the natural resource industry. *Intl. Trans. in Op. Res.* **19** 39–62, 2012.

Brunberg, T. and Arlinger, J. Vad kostar det att sortera virket i skogen? (What does it cost to sort timber at the stump?) *Available from: http://www.skogforsk.se/upload/Dokument/Resultat/2001-03.pdf*, 2001.

Dems, A., Rousseau, L.-M., and Frayret, J.-M. Effects of different cut-to-length harvesting structures on the economic value of a wood procurement planning problem. *Ann. Oper. Res.* **DOI 10.1007/s10479-013-1336-1**, 2013.

Dems, A., Rousseau, L.-M., and Frayret, J.-M. Annual timber procurement planning with bucking decisions. *Technical Report CIRRELT-2014-20, CIRRELT*, 2014.

El Hachemi, N., Gendreau M., and Rousseau, L.-M. A hybrid constraint programming approach to the log-truck scheduling problem. *Annals of Operations Research.* **184** 163–178, 2011.

Flisberg, P., Forsberg, M., and Rönnqvist, M. Optimization based planning tools for routing of forwarders at harvest area. *Canadian Journal of Forest Research.* **37(11)** 2153–2163, 2007.

Gingras, C., Cordeau J.F., Laporte G. Un algorithme de minimisation du transport à vide appliqué à l'industrie forestière. *Information Systems and Operational Research.* **45(1)** 41-47, 2007.

Gingras, J.-F. and Favreau, J. Incidence du triage sur la productivité des systèmes par bois tronçonnés. *Avantage, Feric.* **3**, 2002.

Hooker, J.N. A Hybrid Method for the Planning and Scheduling. *Constraints* 10(4)385–401, 2005.

Kivinen, V.P. Design and testing of stand-specific bucking instructions for use on modern cut-to-length harvesters. *Dissertationes Forestales 37, https://helda.helsinki.fi/bitstream/handle/10138/20652/designan.pdf?sequence=2*, 2007.

Kivinen, V.P. A forest-level genetic algorithm based control system for generating stand-specific log demand distributions. *Canadian Journal of Forest Research.* **36(7)** 1705–1722, 2006.

Laroze, A.J. Development and comparison of stand level bucking optimization methods. *PhD Thesis. Oregon State University*, 1993.

Milano, M. M. Constraint and integer programming. *Kluwer Academic Publishers.*, 2004.

Rönnqvist, M. Optimization in forestry, *Math. Prog.* **97** 267–284, 2003.

Sakkout, H.E., and M. Wallace. Probe backtrack search for minimal perturbation in dynamic scheduling. *Constraints* **5(4)** 359–388, 2000.

Weintraub, A., Romero, C., Björndal, T., Epstein, R. eds. Handbook of Operations Research in Natural Resources. *Springer, New York*, 2007.